AD/A-001 533

MULTIPLE ROUND HIT PROBABILITIES ASSOCIATED WITH COMBAT VEHICLE FIRE CONTROL GUN SYSTEMS

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Philadelphia, Pennsylvania

May 1974

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
R-3011

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER
AD/A-001 535

4. TITLE (and Subtitle)
MULTIPLE ROUND HIT PROBABILITIES ASSOCIATED WITH COMBAT VEHICLE FIRE CONTROL GUN SYSTEMS

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
LOUIS R. CERRATO
KENNETH R. PFLEGER

8. CONTRACT OR GRANT NUMBER(s)
AMCMS Code: 662603.12.22500.03
DA Project: 1W626603A005

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Frankford Arsenal
ATTN: SARFA-FCA-W
Philadelphia, PA 19137

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
ARMCOM

12. REPORT DATE
May 1974

13. NUMBER OF PAGES
52

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Multiple Round Hit Probability
Combat Vehicle Fire Control
Automatic Cannon
Average Number of Rounds to hit
Correlated Bursts

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report describes a method for evaluating the hit probability associated with multiple round or burst fire weapons when used on combat vehicles against ground targets. The problem of correlation between consecutive bursts is also addressed and some hit probability expressions are derived. Tables and curves for representative calculations are presented in an Appendix.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ACCESSION for		
NTIS	White Section	X
DDC	Buff Section	[ ]
UNANNOUNCED		[ ]
JUSTIFICATION		
BY		
DISTRIBUTION		
Dist.		
A		

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INTRODUCTION

This report describes analytical methods which have been formulated and used in the evaluation of fire control systems for both large caliber and automatic cannon weapons used on combat vehicles against ground targets. Current requirements for high performance automatic cannon systems have made this type of analysis necessary.

DISCUSSION

Classification of Errors

Combat vehicle fire control errors[1] can generally be divided into 4 classifications: fixed biases, occasion-to-occasion biases, burst-to-burst biases, and round-to-round errors. An occasion is one tactical engagement between vehicles. A burst is a series of rounds fired in rapid succession with the same point of aim.

Fixed biases, as their name implies, are constant at a given range. Gravity drop-off, drift of spin stabilized projectiles, and parallax error (due to sight offset from weapon) are examples of fixed biases. Another source which may contribute to this type of error is fire control equipment which is damaged or out of adjustment. In a moderately sophisticated, well maintained fire control system, correction is made for all fixed biases.

Occasion-to-occasion biases are errors which change from time-to-time, but change so slowly that they can be considered constant over the period of an engagement. Errors due to vehicle cant, wind velocity, air density and temperature changes are considered to be occasion-to-occasion biases. These biases vary from engagment to engagement in a random fashion. In general, these errors are functions of weapon system parameters (ammunition type, muzzle velocity, etc.) the geometry of the tactical situation (range) and the firing conditions.

[1]For a more complete discussion of combat vehicle error sources refer to: K. Pfleger amd R. Bibbero, "The Evaluation of Combat Vehicle Fire Control/ Gunnery Systems," Frankford Arsenal Report R-1937, Sep 1969.

Burst-to-burst biases are random and generally have different values for each burst fired during an engagement. If a gunner lays the reticle onto the target before each burst of fire from an automatic cannon, then the laying error is a burst-to-burst type of error.

Round-to-round errors are random taking on different values for each round, so that no correction can be introduced by the fire control system. Ammunition dispersion, (due to number and size of projectile propellent grains, fit of projectile to case, orientation of the round in the chamber, etc.) must be treated as a round-to-round error.

Assumptions

The following assumptions are made in analyzing the problem of multi-round hit probabilities:

1. V is a normally distributed random variable * which represents the component of fire control error that remains constant during the period of time (several minutes) of a single engagement. The mean of V is the fixed bias of the system; and the standard deviation of V is the standard deviation of the occasion-to-occasion biases.

2. B is a normally distributed random variable which represents the component of error that remains constant during a burst but has a different realization for each burst. B has zero mean.

3. R is a normally distributed random variable which represents the round-to-round error. R has a zero mean.

4. V, B, and R are mutually independent.

5. The horizontal and vertical components of the errors, represented by (V_x, V_y), (B_x, B_y) and (R_x, R_y) are uncorrelated.

Single Shot Hit Probabilities

The following expressions give the single shot hit probability in terms of the variables defined above (the round fired is considered to be a single burst of one round):

* Throughout this report random variables are represented by upper case letters (e.g., V, B and R) and realizations of these random variables by the corresponding lower case letters (e.g., v, b and r).

The total horizontal error $\varepsilon_x = V_x + B_x + R_x$ is normally distributed with mean $E\{\varepsilon_x\} = \mu_x$ and variance

$$\mathrm{Var}\{\varepsilon_x\} = E\{(\varepsilon_x - \mu_x)^2\} = \sigma_x^2 = \gamma_x^2 + \psi_x^2 + \nu_x^2$$

where $\mu_x = E\{V_x\}$ - fixed bias

$\gamma_x^2 = \mathrm{Var}\{V_x\}$ - variance of occasion- to-occasion biases

$\psi_x^2 = \mathrm{Var}\{B_x\}$ - variance of burst-to-burst biases

$\nu_x^2 = \mathrm{Var}\{R_x\}$ - variance of round-to-round errors

For the total vertical error, replace the subscript x with the subscript y in the above equations.

If the target is rectangular with dimensions ω_x by ω_y (in the same units as the errors) then the single shot hit probability P{H} is given by

$$P\{H\} = \left[\frac{1}{\sqrt{2\pi}\sigma_x}\int_{-\frac{\omega_x}{2}}^{\frac{\omega_x}{2}} e^{-\frac{1}{2}\frac{(s-\mu_x)^2}{\sigma_x^2}}\,ds\right]\cdot\left[\frac{1}{\sqrt{2\pi}\sigma_y}\int_{-\frac{\omega_y}{2}}^{\frac{\omega_y}{2}} e^{-\frac{1}{2}\frac{(t-\mu_y)^2}{\sigma_y^2}}\,dt\right] \qquad (1)$$

Multiple Round

If a burst of N rounds[2] is fired at a target, the probability of getting exactly K hits, P{K/N}, must be determined. If each round were independent of all others fired then this probability would be

$$P\{K/N\} = \binom{N}{K}\left[\dot{P}\{H\}\right]^K\left[1-\dot{P}\{H\}\right]^{N-K}$$

where $\dot{P}\{H\}$ is the single shot hit probability and $\binom{N}{K} = \frac{N!}{K!(N-K)!}$ (binomial coefficients).

---

[2] For specific treatment of the two-round problem refer to: R.H. Peterson, "Hit Probabilities Associates with Two Successive Rounds," BRL Memorandum report No. 1714, Aberdeen Proving Grounds, June 1966.

The rounds are not, in general, independent since part of the total error is shared by all rounds fired during an engagement; another part is shared by all rounds within the burst; and yet another part changes from round-to-round.

If we measure the cartesian coordinates of each round as it passes through a vertical plane containing the target (with the center of the target as the origin) and if the coordinates are designated by (S,T), then

$$S = V_x + B_x + R_x \qquad (2)$$

$$T = Y_y + B_y + R_y$$

Each round in a particular burst will have the same realizations of the bias random variables, V and B, but different realizations of the round-to-round random variable, R. Letting the horizontal bias, $V_x + B_x$, be represented by X and the vertical bias, $V_y + B_y$, be Y; Equations 2 become

$$S = X + R_x$$

$$T = Y + R_y \qquad (3)$$

If realizations of X and Y are specified for a particular burst, that is, if X = x and Y = y, then S and T have means x and y and variances ν_x^2 and ν_y^2 respectively. The conditional probability of getting a hit on the target with any round of the burst given the realizations of X and Y is

$$P\left\{-\frac{\omega_x}{2} \le S \le \frac{\omega_x}{2}, -\frac{\omega_y}{2} \le T \le \frac{\omega_y}{2} \,\middle|\, X=x, Y=y\right\} = \left[\frac{1}{\sqrt{2\pi}\nu_x}\int_{-\frac{\omega_x}{2}}^{\frac{\omega_x}{2}} e^{-\frac{1}{2}\frac{(s-x)^2}{\nu_x^2}}\,ds\right] \qquad (4)$$

$$\cdot \left[\frac{1}{\sqrt{2\pi}\nu_y}\int_{-\frac{\omega_y}{2}}^{\frac{\omega_y}{2}} e^{-\frac{1}{2}\frac{(t-y)^2}{\nu_y^2}}\,dt\right] = P\left\{-\frac{\omega_x}{2} \le S \le \frac{\omega_x}{2} \,\middle|\, X=x\right\} \cdot P\left\{-\frac{\omega_y}{2} \le T \le \frac{\omega_y}{2} \,\middle|\, Y=y\right\}$$

To simplify the notation, the above expression is rewritten as

$$P\{H \mid x,y\} = P\{H_x \mid x\} \; P\{H_y \mid y\}$$

Furthermore, if realizations of X and Y are specified, each round within the burst is independent of the others. The probability of hitting the target with two particular rounds given x and y, is $P\{H|x,y\}^2$

In general, the probability of getting exactly K hits of an N round burst, given x and y, the components of the total bias for that burst is

$$P\{K/N|x,y\} = \binom{N}{K}\left[P\{H|x,y\}\right]^K\left[1-P\{H|x,y\}\right]^{N-K} \tag{5}$$

Multiplying the above expression by the probability density function for the variables X and Y, p(x,y), and integrating over all possible values of bias yields the required results,

$$P\{K/N\} = \binom{N}{K}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[P\{H|x,y\}\right]^K\left[1-P\{H|x,y\}\right]^{N-K} p(x,y)\; dx\; dy \tag{6}$$

The expression for p(x,y) can be obtained since X and Y are normally distributed independent random variables with

$$E\{X\} = E\{V_x + B_x\} = \mu_x$$

$$\mathrm{Var}\{X\} = \mathrm{Var}\{V_x + B_x\} = \eta_x^2 = \gamma_x^2 + \psi_x^2$$

and similar expressions for E {Y} and Var {Y}. The joint density function for X and Y is then given by

$$p(x,y) = \frac{1}{2\pi\eta_x\eta_y}\, e^{-\frac{1}{2}\left[\frac{(x-\mu_x)^2}{\eta_x^2} + \frac{(y-\mu_y)^2}{\eta_y^2}\right]} \tag{7}$$

The multiple integral (6) with the integrand made up of terms from Equation 4 and Equation 7 can be evaluated numerically using either Simpson's rule or a Gaussian quadrature formula. The two single integrals which make up $P\{H|x,y\}$ are usually available as library subroutines in a digital computer system.

An alternate method of evaluating $P\{K/N\}$ is to expand $\left[1-P\{H|x,y\}\right]^{N-K}$ by use of the binomial theorem.

$$\left[1-P\{H|x,y\}\right]^{N-K} = \sum_{j=0}^{N-K} (-1)^j \binom{N-K}{j} \left[P\{H|x,y\}\right]^j$$

$$= \sum_{j=0}^{N-K} (-1)^j \binom{N-K}{j} \left[P\{H_x|x\}\right]^j \cdot \left[P\{H_y|y\}\right]^j$$

Substituting this into (6) interchanging summation and integration, and letting $n=j+K$ yields.

$$P\{K/N\} = \binom{N}{K} \sum_{n=K}^{N} (-1)^{n-K} \binom{N-K}{n-K} P\{n/n\} \tag{8}$$

where

$$P\{n/n\} = \left[\frac{1}{\sqrt{2\pi}\eta_x} \int_{-\infty}^{\infty} \left[P\{H_x|x\}\right]^n e^{-\frac{1}{2}\frac{(x-\mu_x)^2}{\eta_x^2}} dx\right] \cdot \left[\frac{1}{\sqrt{2\pi}\eta_y} \int_{-\infty}^{\infty} \left[P\{H_y|y\}\right]^n e^{-\frac{1}{2}\frac{(y-\mu_y)^2}{\eta_y^2}} dy\right] \tag{9}$$

$P\{^n/_n\}$ represents the probability of achieving exactly n hits of an n round burst. $P\{1/1\}$ is the single shot hit probability, $P\{H\}$. Then the following matrix equation can be written for $P\{K/N\}$ in terms of $P\{^n/n\}$. Note that $P\{^0/0\} \equiv 1$.

$$
\begin{bmatrix} P\{0/N\} \\ P\{1/N\} \\ P\{2/N\} \\ \vdots \\ \vdots \\ P\{N/N\} \end{bmatrix}
=
\begin{bmatrix}
\binom{N}{0} & 0 & 0 & \cdots & 0 & 0 \\
0 & \binom{N}{1} & 0 & \cdots & 0 & 0 \\
0 & 0 & \binom{N}{2} & 0 \cdots & & 0 \\
\vdots & & & \ddots & & \vdots \\
\vdots & & & & \ddots & \\
0 & 0 & \cdots & & 0 & \binom{N}{N}
\end{bmatrix}
\begin{bmatrix}
\binom{N}{0} & -\binom{N}{1} & \binom{N}{2} & -\binom{N}{3} & \cdots & (-1)^{N}\binom{N}{N} \\
0 & \binom{N-1}{0} & -\binom{N-1}{1} & \binom{N-1}{2} & \cdots & (-1)^{N-1}\binom{N-1}{N-1} \\
0 & 0 & \binom{N-2}{0} & -\binom{N-2}{1} & \cdots & (-1)^{N-2}\binom{N-2}{N-2} \\
\vdots & \vdots & & \ddots & & \vdots \\
\vdots & \vdots & & & \ddots & \\
0 & 0 & \cdots & \cdots & \cdots & \binom{0}{0}
\end{bmatrix}
\begin{bmatrix} 1 \\ P\{1/1\} \\ P\{2/2\} \\ \vdots \\ P\{n/n\} \\ \vdots \\ P\{N/N\} \end{bmatrix}
\qquad (10)
$$

The problem thus reduces to an evaluation of the quantities $P\{n/n\}, n = 0,N$, defined by Equation 9 rather than the computationally more complex expression of Equation 6. The difficulty with this method is that the elements of the matrices become large as N increases and, since the signs alternate in one matrix, small errors in the evaluation of $P\{n/n\}$ may cause large errors in $P\{K/N\}$. $N \leq 20$ has been tried with excellent results.

Probability of at Least L Hits of N Rounds Fired

The performance specification of an automatic cannon system may be stated in terms of achieving a certain probability of getting at least L hits of an N round burst. If the quantities $P\{K/N\}$ are known, this probability is easily calculated.

$$
P\ \{\text{at least L hits of N rounds}\} = \sum_{K=L}^{N} P\{K/N\} \qquad (11)
$$

Tables and plots of these probabilities for several values of L and N can be found in the Appendix.

Since $\sum_{K=0}^{N} P\{K/N\} = 1$, Equation 11 can be rewritten as

$$
P\{\text{at least L hits of N rounds}\} = 1 - \sum_{K=0}^{L-1} P\{K/N\} \qquad (11a)
$$

so if $L \leq \frac{1}{2}N$ Equation 11a reduces the number of terms that have to be summed.
Specifically if $L = 1$ we get

$$
\text{Probability}\{\text{at least 1 hit of N rounds}\} = 1 - P\{0/N\} \qquad (12)
$$

Expected Number of Hits

The expected number of rounds that hit a target, E_H, of a burst of N rounds is given by

$$E_H = \sum_{K=0}^{N} K \; P\{K/N\}$$

using the expression for $P\{K/N\}$ of Equation 6, E_H becomes

$$E_H = \sum_{K=0}^{N} K \binom{N}{K} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\left[\{H|x,y\}\right]^K \left[1-P\{H|x,y\}\right]^{N-K} p(x,y)dxdy$$

Interchanging integration and summation yields

$$E_H = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[1-P\{H|x,y\}\right]^N \left[\sum_{K=0}^{N} K \cdot \binom{N}{K} \cdot \left(\frac{P\{H|x,y\}}{1-P\{H|x,y\}}\right)^K\right] p(x,y)dxdy$$

Using the binomial theorem, $(1+r)^N = \sum_{K=0}^{N} \binom{N}{K} r^K$

differentiating this expression with respect to r, and multiplying by r yields

$$Nr(1+r)^{N-1} = \sum_{K=0}^{N} K \binom{N}{K} r^K$$

letting $r = \dfrac{P\{H|x,y\}}{1-P\{H|x,y\}}$, E_H can be written as

$$E_H = N\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\{H|x,y\} p(x,y)dxdy$$

$$= NP\{1/1\}$$

$$= NP\{H\} \qquad (13)$$

The expected number of hits on a target of a burst of N rounds is equal to the single shot hit probability multiplied by N. Values of $P\{H\}$ are tabulated in the Appendix.

Variance of the Number of Hits

The variance of the number of rounds that hit a target in a burst of N rounds is

$$\sigma_H^2 = \sum_{K=0}^{N} (K-E_H)^2 \; P\{K/N\}$$

which can be written as

$$\sigma_H^2 = -E_H^2 + \sum_{K=0}^{N} K^2 P\{K/N\}$$

Substituting the expression of Equation 6 for $P\{K/N\}$, σ_H^2 becomes

$$\sigma_H^2 = -E_H^2 + \sum_{K=0}^{N} K^2 \binom{N}{K} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[P\{H|x,y\}\right]^K \left[1-P\{H|x,y\}\right]^{N-K} p(x,y)dxdy$$

Interchanging integration and summation

$$\sigma_H^2 = -E_H^2 + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[1-P\{H|x,y\}\right]^N \left[\sum_{K=0}^{N} K^2 \binom{N}{K} \left(\frac{P\{H|x,y\}}{1-P\{H|x,y\}}\right)^K\right] p(x,y)dxdy$$

Again using the binomial theorem, $(1+r)^N = \sum_{K=0}^{N} \binom{N}{K} r^K$

differentiating twice with respect to r and multiplying by r^2 yields

$$Nr(1+Nr)(1+r)^{N-2} = \sum_{K=0}^{N} K^2 \binom{N}{K} r^K$$

Letting $r = \frac{P\{H|x,y\}}{1-P\{H|x,y\}}$, σ_H^2 becomes

$$\sigma_H^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[NP\{H|x,y\}+N(N-1)(P\{H|x,y\})^2\right] p(x,y)\,dxdy - E_H^2$$

$$= N\left[P\{1/1\}-P\{2/2\}\right] + N^2\left[P\{2/2\}-(P\{1/1\})^2\right]$$

Letting $P\{HH\}=P\{2/2\}$, that is, the probability of getting exactly two hits of two rounds fired,

$$\sigma_H^2 = N\left[P\{H\} - P\{HH\}\right] + N^2\left[P\{HH\} - (P\{H\})^2\right] \tag{14}$$

Values of $P\{H\}$ and $P\{HH\}$ are tabulated in the Appendix.

Average Number of Rounds to Achieve a Hit

In this section an expression is derived for the average number of rounds which must be fired at a target before the first hit is achieved. It is assumed that no attempt is made to sense miss distance or to make corrections between rounds.

When a series of rounds is fired at a target, the probability that the first hit occurs on the Kth round is

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\{H|x,y\}\left[1-P\{H|x,y\}\right]^{K-1} p(x,y)\,dxdy$$

The average number of rounds which must be fired before achieving a hit is

$$E_R = \sum_{K=1}^{\infty} K \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P\{H|x,y\}\left[1-P\{H|x,y\}\right]^{K-1} p(x,y)\,dxdy$$

Interchanging summation and integration and making use of the relationship

$$\frac{1}{(1-r)^2} = \sum_{K=1}^{\infty} Kr^{K-1} \qquad |r| < 1$$

yields the required expression for the expected number of rounds to hit the target for the first time when $r = [1-P\{H|x,y\}]$

$$E_R = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{P\{H|x,y\}} p(x,y)dxdy$$

$$E_R = \left[\frac{1}{\sqrt{2\pi}\eta_x} \int_{-\infty}^{\infty} \frac{1}{P\{H_x|x\}} e^{-\frac{1}{2}\frac{(x-\mu_x)^2}{\eta_x^2}} dx\right]\left[\frac{1}{\sqrt{2\pi}\eta_y} \int_{-\infty}^{\infty} \frac{1}{P\{H_y|y\}} e^{-\frac{1}{2}\frac{(y-\mu_y)^2}{\eta_y^2}} dy\right] \quad (15)$$

The integrals in (15) diverage for $\eta > \nu$, that is, E_R is infinite. For $\eta \leq \nu$ the integrals converge and the values of E_R are tabulated and plotted in the Appendix.

Correlation Between Bursts

When two burst are fired in succession at the same target, the events are not generally independent. There is a component of bias error, which is common to both bursts, and another component which is, in general, different for each burst.

The horizontal and vertical components of the total bias for the first burst are

$$X_1 = V_x + B_{x1}$$

$$Y_1 = V_y + B_{y1}$$

For the second burst the components of bias are

$$X_2 = V_x + B_{x2}$$

$$Y_2 = V_y + B_{y2}$$

Since the components B_{x1}, B_{x2}, B_{y1} and B_{y2} are mutually independent burst-to-burst random variables, it follows that (X_1-V_x), (X_2-V_x), (Y_1-V_y) and $(Y_2 - V_y)$ are mutally independent with the following conditional probability density function $f(x_1, y_1, x_2, y_2 | V_x = v_x, V_y = v_y)$

$$= \frac{1}{(2\pi\psi_x\psi_y)^2} e^{-\frac{1}{2}\left[\frac{(x_1-v_x)^2}{\psi_x^2} + \frac{(y_1-v_y)^2}{\psi_y^2} + \frac{(x_2-v_x)^2}{\psi_x^2} + \frac{(y_2-v_y)^2}{\psi_y^2}\right]} \quad (16)$$

and the probability density function for the occasion-to-occasion biases is given by

$$g(v_x,v_y)=\frac{1}{2\pi\gamma_x\gamma_y}\,e^{-\frac{1}{2}\left[\frac{(v_x-\mu_x)^2}{\gamma_x^2}+\frac{(v_y-\mu_y)^2}{\gamma_y^2}\right]} \tag{17}$$

The probability of achieving exactly K_1 hits of N_1 rounds fired in the first burst **and** K_2 hits of N_2 rounds fired in the second burst is

$$P\{K_1/N_1,K_2/N_2\}=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\binom{N_1}{K_1}\left[P\{H|x_1,y_1\}\right]^{K_1}\left[1-P\{H|x_1,y_1\}\right]^{N_1-K_1}$$

$$\binom{N_2}{K_2}\left[P\{H|x_2,y_2\}\right]^{K_2}\cdot\left[1-P\{H|x_2,y_2\}\right]^{N_2-K_2}$$

$$f(x_1,y_1,x_2,y_2|v_x,v_y)g(v_x,v_y)\;dx_1dy_1dx_2dy_2dv_xdv_y \tag{18}$$

Integrating on the variables v_x and v_y, the above expression can be written

$$P\{K_1/N_1,K_2/N_2\}=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\binom{N_1}{K_1}\left[P\{H|x_1,y_1\}\right]^{K_1}\left[1-P\{H|x_1,y_1\}\right]^{N_1-K_1}\binom{N_2}{K_2}$$

$$\left[P\{H|x_2,y_2\}\right]^{K_2}\cdot\left[1-P\{H|x_2,y_2\}\right]^{N_2-K_2}h(x_1,y_1,x_2,y_2)dx_1dy_1dx_2dy_2 \tag{19}$$

where

$$h(x_1,y_1,x_2,y_2)=\frac{1}{2\pi\eta_x^2\sqrt{1-\rho_x^2}}\,e^{-\frac{1}{2(1-\rho_x^2)}\left[\frac{(x_1-\mu_x)^2-2\rho_x(x_1-\mu_x)(x_2-\mu_x)+(x_2-\mu_x)^2}{\eta_x^2}\right]}$$

$$\cdot\frac{1}{2\pi\eta_y^2\sqrt{1-\rho_y^2}}\,e^{-\frac{1}{2(1-\rho_y^2)}\left[\frac{(y_1-\mu_y)^2-2\rho_y(y_1-\mu_y)(y_2-\mu_y)+(y_2-\mu_y)^2}{\eta_y^2}\right]} \tag{20}$$

and $\rho_x=\frac{\gamma_x^2}{\gamma_x^2+\psi_x^2}$, $\rho_y=\frac{\gamma_y^2}{\gamma_y^2+\psi_y^2}$

That is, the biases are jointly normally distributed with a correlation coefficient which depends on the relative size of the variance of the burst-to-burst error and the variance of the occasion-to-occasion error. If $\rho_x = \rho_y = 0$ the bursts are independent.

Equation 19 can be simplified by making use of the following relationships:

1. From Equation 20 $h(x_1,y_1,x_2,y_2)$ can be written in the form

$$h(x_1,y_1,x_2,y_2) = h_x(x_1,x_2)\cdot h_y(y_1,y_2)$$

2. From Equation 4

$$P\{H|x_1,y_1\} = P\{H_x|x_1\}\cdot P\{H_y|y_1\}$$

$$P\{H|x_2,y_2\} = P\{H_x|x_2\}\cdot P\{H_y|y_2\}$$

3. By use of the binomial theorem

$$\left[1-P\{H|x_1,y_1\}\right]^{N_1-K_1} = \sum_{i=0}^{N_1-K_1} (-1)^i \binom{N_1-K_1}{i} \left[P\{H_x|x_1\}\right]^i \left[P\{H_y|y_1\}\right]^i$$

$$\left[1-P\{H|x_2,y_2\}\right]^{N_2-K_2} = \sum_{j=0}^{N_2-K_2} (-1)^j \binom{N_2-K_2}{j} \left[P\{H_x|x_2\}\right]^j \left[P\{H_y|y_2\}\right]^j$$

By substituting the above expressions into Equation 19, interchanging summation and integration operations and letting $n=i+K_1$ and $m=j+K_2$, the probability of achieving exactly K_1 hits of N_1 rounds fired in the first burst and K_2 hits of N_2 rounds fired in the second burst becomes

$$P\{K_1/N_1,K_2/N_2\} = \binom{N_1}{K_1}\binom{N_2}{K_2} \sum_{m=K_2}^{N_2} \sum_{n=K_1}^{N_1} (-1)^{n+m-K_1-K_2} \binom{N_1-K_1}{n-K_1}\binom{N_2-K_2}{m-K_2} P\{n/n,m/m\} \quad (21)$$

where

$$P\{n/n,m/m\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[P\{H_x|x_1\}\right]^n \left[P\{H_x|x_2\}\right]^m h_x(x_1,x_2)dx_1,dx_2$$

$$\times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[P\{H_y|y_1\}\right]^n \left[P\{H_y|y_2\}\right]^m h_y(y_1,y_2)dy_1,dy_2 \qquad (22)$$

This procedure reduces the complexity of the calculations. However, as noted previously, the appearance of the binomial coefficients in the expression puts a practical limit on the size of N_1 and N_2.

APPENDIX

Tabulation of Numerical Calculations of Hit Probabilities

Tables I through XI give the probability of achieving at least L hits of N rounds fired at a target, for various values of L and N, as a function of the total bias standard deviation, η, and the round-to-round standard deviation ν. Table XII gives the single shot hit probability, $P\{H\}$ (or $P\{1/1\}$), and Table XIII gives the probability of achieving two hits of two rounds fired at a target, $P\{HH\}$ (or $P\{2/2\}$). Table XIV gives the average number of rounds to achieve a hit, E_R. In all of the tables η varies from 0 to 20 mils and represents both the vertical and horizontal components of bias (i.e., $\eta_x = \eta_y = \eta$). ν varies from 0 to 6 mils, also equal in both dimensions ($\nu_x = \nu_y = \nu$). These tabulated data have been plotted in Figures 1 through 14. The figures have been numbered to correspond with the tables.

All calculations were made for a target size of 7.5' x 7.5' at 1000 meters. To use the tables and figures for other size targets at other ranges the following equations should be used:

$$\eta = \frac{7.5}{T} \cdot \frac{R}{1000} \; \eta(R,T)$$

$$\nu = \frac{7.5}{T} \cdot \frac{R}{1000} \; \nu(R,T)$$

where η and ν are tabulated values and $\eta(R,T)$ and $\nu(R,T)$ are the errors at a range R (in meters) against a target measuring T x T (in feet).

As an example, suppose we wish to know the probability of achieving at least 1 hit of 2 rounds fired at a 15' x 15' target at a range of 1500 meters, given that the bias standard deviation is 0.8 mils and the round-to-round standard deviation 0.4 mils. Applying the above equations we get:

$$\eta = \frac{7.5}{15} \times \frac{1500}{1000} \times 0.8 = 0.6$$

$$\nu = \frac{7.5}{15} \times \frac{1500}{1000} \times 0.4 = 0.3$$

Consulting Table I for $\eta = 0.6$ and $\nu = 0.3$ the probability of at least 1 hit of 2 rounds fired is 0.9187.

TABLE I
Probability of Achieving at Least One Hit of a Two Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION μ IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	.9999	.9984	.9896	.9271	.8160	.6934	.5325	.3490	.1741	.1021	.0667	.0468
0.1	1.000	1.000	1.000	1.000	.9999	.9980	.9883	.9242	.8129	.6909	.5309	.3483	.1740	.1021	.0666	.0468
0.2	1.000	1.000	1.000	1.000	.9995	.9959	.9832	.9149	.8037	.6835	.5263	.3463	.1734	.1019	.0665	.0467
0.3	.9998	.9998	.9997	.9989	.9964	.9889	.9717	.8982	.7881	.6713	.5186	.3429	.1726	.1016	.0664	.0467
0.4	.9928	.9944	.9938	.9909	.9845	.9720	.9501	.8731	.7662	.6545	.5082	.3382	.1714	.1012	.0662	.0466
0.5	.9606	.9682	.9694	.9656	.9564	.9405	.9161	.8397	.7385	.6335	.4952	.3324	.1699	.1006	.0660	.0465
0.6	.8981	.9140	.9202	.9187	.9101	.8942	.8704	.7988	.7059	.6091	.4801	.3255	.1680	.1000	.0657	.0463
0.8	.7300	.7578	.7749	.7825	.7814	.7726	.7563	.7032	.6310	.5531	.4449	.3090	.1636	.0984	.0650	.0460
1.0	.5710	.6015	.6236	.6377	.6441	.6436	.6363	.6033	.5520	.4928	.4059	.2901	.1582	.0964	.0642	.0455
1.2	.4463	.4747	.4972	.5137	.5242	.5290	.5283	.5111	.4770	.4338	.3662	.2697	.1520	.0941	.0631	.0450
1.5	.3162	.3395	.3592	.3751	.3872	.3954	.3997	.3967	.3800	.3545	.3096	.2385	.1418	.0901	.0613	.0441
2.0	.1932	.2091	.2232	.2355	.2458	.2541	.2603	.2659	.2628	.2531	.2316	.1906	.1237	.0825	.0577	.0422
3.0	.0912	.0993	.1068	.1137	.1198	.1252	.1298	.1364	.1392	.1388	.1342	.1208	.0906	.0665	.0494	.0376
4.0	.0524	.0572	.0617	.0659	.0697	.0731	.0762	.0810	.0838	.0850	.0844	.0798	.0659	.0522	.0411	.0326
5.0	.0339	.0370	.0400	.0427	.0453	.0477	.0498	.0532	.0554	.0567	.0571	.0555	.0488	.0410	.0338	.0278
6.0	.0237	.0259	.0280	.0299	.0317	.0334	.0350	.0375	.0392	.0403	.0409	.0405	.0370	.0324	.0278	.0236
8.0	.0134	.0146	.0158	.0170	.0180	.0190	.0199	.0214	.0225	.0232	.0238	.0240	.0229	.0211	.0191	.0171
10.	.0086	.0094	.0102	.0109	.0116	.0122	.0128	.0138	.0145	.0150	.0155	.0157	.0154	.0146	.0136	.0126
15.	.0038	.0042	.0045	.0049	.0052	.0054	.0057	.0062	.0065	.0067	.0070	.0072	.0072	.0071	.0068	.0066
20.	.0022	.0024	.0026	.0027	.0029	.0031	.0032	.0035	.0037	.0038	.0039	.0041	.0041	.0041	.0040	.0039

TABLE II

Probability of Achieving at Least One Hit of a Three Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION μ IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	.9999	.9989	.9803	.9211	.8302	.6803	.4748	.2495	.1492	.0983	.0693
0.1	1.000	1.000	1.000	1.000	1.000	.9999	.9987	.9791	.9191	.8282	.6787	.4739	.2492	.1491	.0983	.0693
0.2	1.000	1.000	1.000	1.000	1.000	.9997	.9977	.9750	.9129	.8219	.6739	.4714	.2485	.1489	.0981	.0693
0.3	.9998	.9999	.9999	.9998	.9994	.9981	.9942	.9665	.9018	.8112	.6659	.4673	.2474	.1484	.0980	.0692
0.4	.9923	.9957	.9965	.9962	.9947	.9914	.9847	.9515	.8851	.7960	.6548	.4616	.2457	.1479	.0977	.0691
0.5	.9606	.9731	.9787	.9802	.9786	.9738	.9650	.9285	.8623	.7762	.6407	.4544	.2436	.1471	.0974	.0689
0.6	.8981	.9234	.9377	.9444	.9454	.9415	.9327	.8966	.8334	.7521	.6240	.4458	.2411	.1462	.0970	.0687
0.8	.7300	.7732	.8038	.8237	.8349	.8387	.8360	.8111	.7608	.6934	.5836	.4251	.2350	.1439	.0960	.0682
1.0	.5710	.6179	.6550	.6829	.7024	.7144	.7197	.7108	.6772	.6262	.5372	.4007	.2275	.1411	.0947	.0675
1.2	.4463	.4899	.5266	.5564	.5795	.5964	.6074	.6116	.5931	.5573	.4883	.3741	.2189	.1378	.0932	.0668
1.5	.3162	.3518	.3833	.4106	.4336	.4523	.4668	.4822	.4793	.4610	.4167	.3327	.2046	.1320	.0905	.0654
2.0	.1932	.2174	.2397	.2601	.2784	.2944	.3083	.3281	.3361	.3334	.3149	.2678	.1792	.1211	.0853	.0626
3.0	.0912	.1036	.1153	.1264	.1368	.1465	.1554	.1703	.1802	.1849	.1843	.1711	.1319	.0978	.0731	.0558
4.0	.0524	.0597	.0667	.0734	.0798	.0859	.0916	.1016	.1090	.1137	.1164	.1134	.0961	.0770	.0609	.0485
5.0	.0339	.0387	.0433	.0477	.0520	.0561	.0600	.0669	.0723	.0760	.0790	.0791	.0712	.0604	.0502	.0414
6.0	.0237	.0270	.0303	.0334	.0364	.0394	.0421	.0472	.0512	.0541	.0567	.0577	.0541	.0478	.0412	.0352
8.0	.0134	.0153	.0171	.0189	.0207	.0224	.0240	.0270	.0294	.0312	.0330	.0342	.0336	.0312	.0284	.0254
10.	.0086	.0098	.0110	.0122	.0133	.0144	.0154	.0174	.0190	.0202	.0214	.0224	.0225	.0216	.0202	.0187
15.	.0038	.0044	.0049	.0054	.0059	.0064	.0069	.0078	.0085	.0091	.0097	.0102	.0105	.0104	.0102	.0098
20.	.0022	.0025	.0028	.0031	.0033	.0036	.0039	.0044	.0048	.0051	.0055	.0058	.0060	.0061	.0060	.0059

TABLE III
Probability of Achieving at Least One Hit of a Five Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9986	.9855	.9480	.8505	.6581	.3802	.2361	.1584	.1129
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9984	.9849	.9469	.8493	.6572	.3798	.2360	.1583	.1129
0.2	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9978	.9828	.9435	.8455	.6545	.3789	.2356	.1582	.1128
0.3	.9998	.9999	1.000	1.000	.9999	.9999	.9996	.9960	.9786	.9375	.8390	.6499	.3772	.2350	.1579	.1126
0.4	.9928	.9968	.998[illegible]	.9987	.9987	.9984	.9975	.9916	.9712	.9282	.8296	.6435	.3750	.2341	.1575	.1124
0.5	.9606	.9778	.9860	.9898	.9913	.9913	.9901	.9821	.9591	.9149	.8173	.6354	.3721	.2329	.1570	.1122
0.6	.8981	.9330	.9533	.9648	.9707	.9732	.9730	.9649	.9411	.8970	.8019	.6256	.3686	.2316	.1563	.1118
0.8	.7300	.7896	.8328	.8631	.8834	.8963	.9036	.9044	.8860	.8474	.7621	.6011	.3600	.2281	.1548	.1110
1.0	.5710	.6359	.6885	.7298	.7613	.7847	.8012	.8170	.8105	.7827	.7125	.5713	.3494	.2238	.1528	.1100
1.2	.4463	.5068	.5589	.6028	.6390	.6681	.6909	.7194	.7254	.7099	.6568	.5375	.3371	.2188	.1504	.1088
1.5	.3162	.3656	.4105	.4506	.4859	.5165	.5426	.5811	.6002	.5999	.5701	.4832	.3164	.2101	.1463	.1066
2.0	.1932	.2268	.2587	.2886	.3165	.3421	.3654	.4044	.4310	.4436	.4392	.3942	.2788	.1932	.1380	.1022
3.0	.0912	.1084	.1252	.1415	.1573	.1726	.1872	.2139	.2357	.2512	.2621	.2558	.2070	.1569	.1187	.0912
4.0	.0524	.0626	.0726	.0825	.0922	.1017	.1110	.1285	.1438	.1557	.1668	.1709	.1517	.1240	.0991	.0793
5.0	.0339	.0405	.0471	.0537	.0601	.0665	.0728	.0849	.0957	.1045	.1136	.1196	.1128	.0975	.0817	.0679
6.0	.0237	.0283	.0330	.0376	.0422	.0468	.0513	.0600	.0679	.0745	.0817	.0875	.0858	.0773	.0673	.0577
8.0	.0134	.0160	.0187	.0213	.0240	.0266	.0292	.0343	.0390	.0431	.0477	.0520	.0534	.0506	.0464	.0417
10.	.0086	.0103	.0120	.0137	.0154	.0171	.0188	.0222	.0253	.0279	.0316	.0341	.0359	.0350	.0331	.0308
15.	.0038	.0046	.0054	.0061	.0069	.0077	.0084	.0099	.0113	.0126	.0140	.0156	.0168	.0169	.0166	.0161
20.	.0022	.0026	.0030	.0034	.0039	0043	.0047	.0056	.0064	.0071	.0079	.0[illegible]88	.0096	.0098	.0098	.0096

TABLE IV
Probability of Achieving at Least Four Hits of a Five Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	.9995	.9857	.9157	.5907	.2888	.1275	.0374	.0059	.0003	.0000	.0000	.0000
0.1	1.000	1.000	1.000	1.000	.9991	.9818	.9062	.5795	.2832	.1254	.0369	.0058	.0003	.0000	.0000	.0000
0.2	1.000	1.000	1.000	.9997	.9951	.9649	.8739	.5472	.2674	.1192	.0355	.0057	.0003	.0000	.0000	.0000
0.3	.9998	.9995	.9982	.9930	.9744	.9216	.8142	.4983	.2443	.1102	.0334	.0055	.0003	.0000	.0000	.0000
0.4	.9928	.9885	.9791	.9595	.9198	.8468	.7318	.4406	.2176	.0995	.0308	.0052	.0003	.0000	.0000	.0000
0.5	.9606	.9472	.9252	.8897	.8342	.7516	.6395	.3818	.1903	.0884	.0280	.0049	.0003	.0000	.0000	.0000
0.6	.8981	.8752	.8428	.7973	.7349	.6525	.5497	.3271	.1648	.0778	.0252	.0045	.0003	.0000	.0000	.0000
0.8	.7300	.6974	.6573	.6083	.5494	.4798	.4004	.2386	.1226	.0594	.0201	.0038	.0003	.0000	.0000	.0000
1.0	.5710	.5385	.5010	.4580	.4093	.3548	.2949	.1765	.0921	.0455	.0159	.0032	.0002	.0000	.0000	.0000
1.2	.4463	.4173	.3852	.3497	.3107	.2682	.2226	.1337	.0705	.0354	.0127	.0026	.0002	.0000	.0000	.0000
1.5	.3162	.2935	.2690	.2427	.2146	.1847	.1531	.0924	.0493	.0251	.0092	.0020	.0002	.0000	.0000	.0000
2.0	.1932	.1781	.1624	.1458	.1284	.1103	.0914	.0554	.0298	.0154	.0058	.0013	.0001	.0000	.0000	.0000
3.0	.0912	.0837	.0760	.0680	.0597	.0512	.0424	.0258	.0140	.0073	.0028	.0007	.0001	.0000	.0000	.0000
4.0	.0524	.0480	.0435	.0389	.0341	.0293	.0242	.0148	.0080	.0042	.0016	.0004	.0000	.0000	.0000	.0000
5.0	.0339	.0310	.0281	.0251	.0220	.0189	.0156	.0095	.0052	.0027	.0011	.0003	.0000	.0000	.0000	.0000
6.0	.0237	.0217	.0196	.0175	.0154	.0132	.0109	.0066	.0036	.0019	.0007	.0002	.0000	.0000	.0000	.0000
8.0	.0134	.0122	.0111	.0099	.0087	.0074	.0062	.0038	.0021	.0011	.0004	.0001	.0000	.0000	.0000	.0000
10.	.0086	.0079	.0071	.0063	.0056	.0048	.0039	.0024	.0013	.0007	.0003	.0001	.0000	.0000	.0000	.0000
15.	.0038	.0035	.0032	.0028	.0025	.0021	.0018	.0011	.0006	.0003	.0001	.0000	.0000	.0000	.0000	.0000
20.	.0022	.0020	.0018	.0016	.0014	.0012	.0010	.0006	.0003	.0002	.0001	.0000	.0000	.0000	.0000	.0000

TABLE V

Probability of Achieving at Least One Hit of a Ten Round Burst

TOTAL BIAS STANDARD DEVIATION η IN MILS	ROUND TO ROUND STANDARD DEVIATION ν IN MILS															
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9998	.9973	.9777	.8831	.6158	.4165	.2917	.2130
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9998	.9972	.9773	.8825	.6154	.4163	.2916	.2130
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9997	.9968	.9761	.8806	.6142	.4157	.2913	.2128
0.3	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9995	.9960	.9739	.8774	.6122	.4147	.2908	.2126
0.4	.9928	.9978	.9992	.9996	.9998	.9998	.9999	.9997	.9988	.9943	.9705	.8727	.6093	.4133	.2901	.2122
0.5	.9606	.9823	.9916	.9956	.9974	.9981	.9985	.9984	.9969	.9912	.9654	.8666	.6057	.4116	.2893	.2117
0.6	.8981	.9429	.9672	.9802	.9871	.9907	.9927	.9939	.9923	.9857	.9582	.8588	.6012	.4095	.2882	.2112
0.8	.7300	.8079	.8629	.9006	.9259	.9427	.9539	.9658	.9686	.9621	.9352	.8379	.5900	.4042	.2856	.2097
1.0	.5710	.6566	.7255	.7795	.8209	.8523	.8759	.9063	.9205	.9215	.8993	.8097	.5759	.3975	.2822	.2079
1.2	.4463	.5265	.5959	.6548	.7039	.7442	.7770	.8243	.8519	.8629	.8513	.7747	.5590	.3895	.2782	.2057
1.5	.3162	.3820	.4426	.4976	.5469	.5904	.6286	.6899	.7326	.7577	.7644	.7128	.5296	.3755	.2710	.2018
2.0	.1932	.2382	.2817	.3234	.3631	.4004	.4353	.4973	.5474	.5842	.6126	.5994	.4735	.3478	.2566	.1938
3.0	.0912	.1142	.1373	.1604	.1833	.2059	.2282	.2710	.3102	.3438	.3803	.4029	.3594	.2860	.2223	.1738
4.0	.0524	.0660	.0799	.0939	.1080	.1222	.1364	.1647	.1919	.2165	.2463	.2736	.2668	.2280	.1868	.1517
5.0	.0339	.0428	.0519	.0612	.0706	.0802	.0899	.1094	.1286	.1465	.1692	.1933	.1999	.1804	.1547	.1303
6.0	.0237	.0299	.0364	.0429	.0496	.0565	.0634	.0775	.0916	.1049	.1223	.1421	.1529	.1436	.1278	.1110
8.0	.0134	.0170	.0206	.0244	.0283	.0322	.0363	.0445	.0529	.0609	.0717	.0848	.0955	.0944	.0884	.0806
10.	.0086	.0109	.0133	.0157	.0182	.0207	.0234	.0288	.0342	.0396	.0468	.0559	.0644	.0655	.0633	.0595
15.	.0038	.0049	.0059	.0070	.0081	.0093	.0105	.0129	.0154	.0178	.0212	.0256	.0302	.0318	.0318	.0312
20.	.0022	.0027	.0033	.0039	.0046	.0052	.0059	.0073	.0087	.0101	.0120	.0145	.0173	.0184	.0188	.0187

TABLE VI
Probability of Achieving at Least Five Hits of a Ten Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	.9998	.9713	.7813	.4860	.1795	.0285	.0011	.0001	.0000	.0000
0.1	1.000	1.000	1.000	1.000	1.000	1.000	.9998	.9683	.7743	.4802	.1774	.0282	.0011	.0001	.0000	.0000
0.2	1.000	1.000	1.000	1.000	1.000	.9999	.9990	.9572	.7526	.4631	.1712	.0275	.0011	.0001	.0000	.0000
0.3	.9998	.9998	.9997	.9596	.9992	.9978	.9931	.9312	.7152	.4362	.1617	.0263	.0010	.0001	.0000	.0000
0.4	.9928	.9932	.9928	.9916	.9886	.9823	.9691	.8837	.6636	.4019	.1498	.0249	.0010	.0001	.0000	.0000
0.5	.9606	.9623	.9619	.9590	.9521	.9394	.9177	.8161	.6024	.3636	.1368	.0232	.0010	.0001	.0000	.0000
0.6	.8981	.9014	.9014	.8972	.8874	.8703	.8434	.7367	.5379	.3247	.1234	.0214	.0009	.0001	.0000	.0000
0.8	.7300	.7353	.7361	.7313	.7200	.7012	.6735	.5772	.4177	.2534	.0985	.0179	.0008	.0001	.0000	.0000
1.0	.5710	.5766	.5776	.5734	.5634	.5471	.5236	.4454	.3219	.1967	.0780	.0148	.0007	.0001	.0000	.0000
1.2	.4463	.4513	.4523	.4489	.4407	.4274	.4083	.3464	.2506	.1541	.0621	.0121	.0006	.0001	.0000	.0000
1.5	.3162	.3202	.3211	.3186	.3126	.3029	.2891	.2449	.1776	.1100	.0451	.0092	.0005	.0000	.0000	.0000
2.0	.1932	.1958	.1964	.1949	.1911	.1851	.1767	.1496	.1088	.0679	.0283	.0060	.0004	.0000	.0000	.0000
3.0	.0912	.0926	.0929	.0921	.0904	.0875	.0835	.0707	.0516	.0324	.0137	.0030	.0002	.0000	.0000	.0000
4.0	.0524	.0532	.0534	.0530	.0520	.0503	.0480	.0407	.0297	.0187	.0080	.0018	.0001	.0000	.0000	.0000
5.0	.0339	.0344	.0345	.0343	.0336	.0325	.0310	.0263	.0192	.0121	.0052	.0012	.0001	.0000	.0000	.0000
6.0	.0237	.0240	.0241	.0239	.0235	.0227	.0217	.0184	.0134	.0085	.0036	.0008	.0001	.0000	.0000	.0000
8.0	.0134	.0136	.0136	.0135	.0133	.0129	.0123	.0104	.0076	.0048	.0021	.0005	.0000	.0000	.0000	.0000
10.	.0086	.0087	.0088	.0087	.0085	.0082	.0079	.0067	.0049	.0031	.0013	.0003	.0000	.0000	.0000	.0000
15.	.0038	.0039	.0039	.0039	.0038	.0037	.0035	.0030	.0022	.0014	.0006	.0001	.0000	.0000	.0000	.0000
20.	.0022	.0022	.0022	.0022	.0021	.0021	.0020	.0017	.0012	.0008	.0003	.0001	.0000	.0000	.0000	.0000

TABLE VII

Probability of Achieving at Least Eight Hits of a Ten Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	.9941	.9265	.4666	.1250	.0259	.0023	.0001	.0000	.0000	.0000	.0000
0.1	1.000	1.000	1.000	1.000	.9999	.9915	.9147	.4513	.1205	.0251	.0023	.0001	.0000	.0000	.0000	.0000
0.2	1.000	1.000	1.000	.9999	.9978	.9756	.8716	.4090	.1085	.0228	.0021	.0001	.0000	.0000	.0000	.0000
0.3	.9998	.9994	.9981	.9935	.9770	.9232	.7883	.3508	.0929	.0198	.0019	.0001	.0000	.0000	.0000	.0000
0.4	.9928	.9874	.9768	.9555	.9122	.8275	.6789	.2905	.0771	.0167	.0016	.0000	.0000	.0000	.0000	.0000
0.5	.9606	.9436	.9173	.8760	.8112	.7117	.5674	.2367	.0631	.0139	.0014	.0000	.0000	.0000	.0000	.0000
0.6	.8981	.8687	.8290	.7745	.6999	.5995	.4688	.1925	.0516	.0116	.0012	.0000	.0000	.0000	.0000	.0000
0.8	.7300	.6878	.6375	.5774	.5057	.4209	.3220	.1301	.0352	.0080	.0009	.0000	.0000	.0000	.0000	.0000
1.0	.5710	.5286	.4812	.4279	.3684	.3021	.2285	.0916	.0250	.0058	.0006	.0000	.0000	.0000	.0000	.0000
1.2	.4463	.4085	.3677	.3234	.2756	.2241	.1685	.0673	.0184	.0043	.0005	.0000	.0000	.0000	.0000	.0000
1.5	.3162	.2865	.2553	.2224	.1879	.1517	.1134	.0452	.0124	.0029	.0003	.0000	.0000	.0000	.0000	.0000
2.0	.1932	.1735	.1533	.1325	.1112	.0892	.0665	.0264	.0073	.0017	.0002	.0000	.0000	.0000	.0000	.0000
3.0	.0912	.0814	.0715	.0614	.0513	.0410	.0304	.0121	.0033	.0008	.0001	.0000	.0000	.0000	.0000	.0000
4.0	.0524	.0467	.0409	.0351	.0292	.0233	.0173	.0069	.0019	.0005	.0001	.0000	.0000	.0000	.0000	.0000
5.0	.0339	.0301	.0264	.0226	.0188	.0150	.0111	.0044	.0012	.0003	.0000	.0000	.0000	.0000	.0000	.0000
6.0	.0237	.0210	.0184	.0158	.0131	.0105	.0077	.0031	.0009	.0002	.0000	.0000	.0000	.0000	.0000	.0000
8.0	.0134	.0119	.0104	.0089	.0074	.0059	.0044	.0017	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000
10.	.0086	.0076	.0067	.0057	.0047	.0038	.0028	.0011	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000
15.	.0038	.0034	.0030	.0025	.0021	.0017	.0012	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20.	.0022	.0019	.0017	.0014	.0012	.0009	.0007	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000

TABLE VIII
Probability of Achieving at Least One Hit of a Twenty Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9995	.9863	.8524	.6595	.4983	.3807
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9995	.9862	.8521	.6593	.4982	.3806
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9994	.9857	.8511	.6586	.4978	.3803
0.3	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9993	.9849	.8496	.6574	.4971	.3799
0.4	.9928	.9984	.9996	.9999	1.000	1.000	1.000	1.000	1.000	.9999	.9991	.9837	.8473	.6558	.4961	.3794
0.5	.9606	.9856	.9947	.9980	.9991	.9996	.9998	.9999	.9999	.9998	.9986	.9819	.8444	.6538	.4949	.3786
0.6	.8981	.9503	.9762	.9884	.9940	.9967	.9980	.9991	.9993	.9992	.9976	.9794	.8408	.6512	.4934	.3777
0.8	.7300	.8228	.8855	.9261	.9517	.9677	.9778	.9882	.9925	.9939	.9922	.9713	.8313	.6448	.4895	.3755
1.0	.5710	.6741	.7554	.8174	.8636	.8974	.9220	.9527	.9686	.9763	.9783	.9575	.8186	.6366	.4847	.3726
1.2	.4463	.5435	.6271	.6971	.7546	.8011	.8382	.8908	.9227	.9412	.9520	.9362	.8026	.6266	.4788	.3691
1.5	.3162	.3964	.4705	.5379	.5980	.6510	.6972	.7711	.8240	.8599	.8892	.8897	.7725	.6085	.4682	.3628
2.0	.1932	.2482	.3021	.3544	.4044	.4518	.4964	.5761	.6424	.6951	.7496	.7831	.7085	.5711	.4464	.3497
3.0	.0912	.1195	.1483	.1776	.2072	.2368	.2663	.3240	.3788	.4287	.4912	.5564	.5592	.4807	.3921	.3164
4.0	.0524	.0691	.0865	.1044	.1227	.1415	.1607	.1994	.2381	.2753	.3258	.3883	.4251	.3898	.3333	.2783
5.0	.0339	.0448	.0563	.0682	.0805	.0932	.1063	.1333	.1608	.1881	.2265	.2782	.3230	.3120	.2785	.2405
6.0	.0237	.0314	.0394	.0479	.0567	.0658	.0752	.0948	.1151	.1354	.1648	.2063	.2491	.2503	.2314	.2059
8.0	.0134	.0178	.0224	.0272	.0323	.0376	.0431	.0546	.0667	.0791	.0973	.1242	.1572	.1660	.1614	.1504
10.	.0086	.0114	.0144	.0175	.0208	.0242	.0278	.0353	.0433	.0515	.0637	.0822	.1065	.1158	.1160	.1116
15.	.0038	.0051	.0064	.0078	.0093	.0108	.0125	.0159	.0195	.0233	.0290	.0377	.0502	.0564	.0587	.0588
20.	.0022	.0029	.0036	.0044	.0052	.0061	.0070	.0090	.0110	.0132	.0164	.0215	.0288	.0328	.0347	.0353

TABLE IX
Probability of Achieving at Least Five Hits of a Twenty Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3.	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9992	.9796	.8077	.3407	.0306	.0032	.0005	.0001
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9991	.9786	.8047	.3389	.0305	.0032	.0005	.0001
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9986	.9751	.7955	.3334	.0301	.0031	.0004	.0001
0.3	.9998	.9999	1.000	1.000	1.000	1.000	1.000	.9998	.9970	.9676	.7795	.3244	.0295	.0031	.0004	.0001
0.4	.9928	.9960	.9976	.9985	.9989	.9990	.9990	.9980	.9914	.9531	.7561	.3125	.0287	.0030	.0004	.0001
0.5	.9606	.9733	.9814	.9864	.9892	.9906	.9910	.9887	.9765	.9285	.7252	.2981	.0278	.0030	.0004	.0001
0.6	.8981	.9226	.9402	.9523	.9602	.9648	.9669	.9647	.9477	.8921	.6375	.2818	.0267	.0029	.0004	.0001
0.8	.7300	.7698	.8016	.8261	.8443	.8570	.8651	.8637	.8499	.7903	.5996	.2464	.0242	.0027	.0004	.0001
1.0	.5710	.6132	.6486	.6774	.7003	.7176	.7301	.7406	.7264	.6732	.5086	.2112	.0217	.0025	.0004	.0001
1.2	.4463	.4850	.5183	.5463	.5693	.5877	.6016	.6159	.6069	.5630	.4259	.1792	.0192	.0023	.0004	.0001
1.5	.3162	.3474	.3748	.3985	.4185	.4351	.4483	.4639	.4601	.4282	.3256	.1397	.0158	.0020	.0003	.0001
2.0	.1932	.2141	.2329	.2495	.2637	.2761	.2862	.2993	.2990	.2798	.2145	.0943	.0114	.0015	.0003	.0001
3.0	.0912	.1018	.1115	.1201	.1277	.1343	.1399	.1476	.1485	.1397	.1081	.0487	.0064	.0009	.0002	.0000
4.0	.0524	.0587	.0644	.0695	.0740	.0780	.0814	.0862	.0870	.0820	.0637	.0290	.0039	.0006	.0001	.0000
5.0	.0339	.0380	.0417	.0451	.0481	.0507	.0530	.0561	.0567	.0536	.0417	.0191	.0026	.0004	.0001	.0000
6.0	.0237	.0265	.0292	.0315	.0336	.0355	.0371	.0394	.0398	.0376	.0293	.0135	.0019	.0003	.0001	.0000
8.0	.0134	.0150	.0165	.0179	.0191	.0201	.0210	.0224	.0226	.0214	.0167	.0077	.0011	.0002	.0000	.0000
10.	.0086	.0096	.0106	.0115	.0122	.0129	.0135	.0144	.0146	.0138	.0107	.0050	.0007	.0001	.0000	.0000
15.	.0038	.0043	.0047	.0051	.0055	.0058	.0060	.0064	.0065	.0061	.0048	.0022	.0003	.0001	.0000	.0000
20.	.0022	.0024	.0027	.0029	.0031	.0032	.0034	.0036	.0037	.0035	.0027	.0013	.0002	.0000	.0000	.0000

TABLE X

Probability of Achieving at Least Ten Hits of a Twenty Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9926	.8078	.3956	.0669	.0020	.0000	.0000	.0000	.0000
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.9912	.7986	.3878	.0654	.0019	.0000	.0000	.0000	.0000
0.2	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9844	.7693	.3654	.0613	.0018	.0000	.0000	.0000	.0000
0.3	.9998	.9998	.9998	.9997	.9995	.9989	.9970	.9616	.7175	.3317	.0555	.0017	.0000	.0000	.0000	.0000
0.4	.9928	.9930	.9928	.9919	.9898	.9854	.9762	.9093	.6468	.2920	.0488	.0015	.0000	.0000	.0000	.0000
0.5	.9606	.9614	.9608	.9582	.9523	.9413	.9224	.8290	.5673	.2518	.0422	.0014	.0000	.0000	.0000	.0000
0.6	.8981	.8995	.8985	.8939	.8842	.8677	.8418	.7351	.4891	.2147	.0362	.0012	.0000	.0000	.0000	.0000
0.8	.7300	.7321	.7303	.7234	.7102	.6895	.6598	.5557	.3578	.1553	.0265	.0009	.0000	.0000	.0000	.0000
1.0	.5710	.5731	.5709	.5638	.5509	.5316	.5049	.4172	.2642	.1142	.0197	.0007	.0000	.0000	.0000	.0000
1.2	.4463	.4480	.4460	.4395	.4282	.4117	.3893	.3182	.1997	.0861	.0149	.0005	.0000	.0000	.0000	.0000
1.5	.3162	.3175	.3158	.3106	.3019	.2894	.2726	.2208	.1376	.0593	.0104	.0004	.0000	.0000	.0000	.0000
2.0	.1932	.1940	.1928	.1893	.1836	.1756	.1649	.1327	.0822	.0354	.0062	.0002	.0000	.0000	.0000	.0000
3.0	.0912	.0916	.0910	.0893	.0864	.0825	.0773	.0619	.0382	.0164	.0029	.0001	.0000	.0000	.0000	.0000
4.0	.0524	.0527	.0523	.0513	.0496	.0473	.0443	.0354	.0218	.0094	.0017	.0001	.0000	.0000	.0000	.0000
5.0	.0339	.0341	.0338	.0331	.0321	.0306	.0286	.0229	.0141	.0061	.0011	.0000	.0000	.0000	.0000	.0000
6.0	.0237	.0238	.0236	.0231	.0224	.0213	.0200	.0159	.0098	.0042	.0008	.0000	.0000	.0000	.0000	.0000
8.0	.0134	.0134	.0133	.0131	.0127	.0121	.0113	.0090	.0055	.0024	.0004	.0000	.0000	.0000	.0000	.0000
10.	.0086	.0086	.0086	.0084	.0081	.0077	.0072	.0058	.0036	.0015	.0003	.0000	.0000	.0000	.0000	.0000
15.	.0038	.0038	.0038	.0037	.0036	.0034	.0032	.0026	.0016	.0007	.0001	.0000	.0000	.0000	.0000	.0000
20.	.0022	.0022	.0021	.0021	.0020	.0019	.0018	.0014	.0009	.0004	.0001	.0000	.0000	.0000	.0000	.0000

TABLE XI

Probability of Achieving at Least Sixteen Hits of a Twenty Round Burst

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.000	.9991	.9537	.3376	.0292	.0014	.0000	.0000	.0000	.0000	.0000	.0000
0.1	1.000	1.000	1.000	1.000	1.000	.9983	.9418	.3186	.0273	.0013	.0000	.0000	.0000	.0000	.0000	.0000
0.2	1.000	1.000	1.000	1.000	.9992	.9877	.8900	.2702	.0226	.0011	.0000	.0000	.0000	.0000	.0000	.0000
0.3	.9998	.9994	.9981	.9941	.9802	.9316	.7810	.2131	.0176	.0009	.0000	.0000	.0000	.0000	.0000	.0000
0.4	.9928	.9868	.9755	.9536	.9090	.8192	.6446	.1631	.0133	.0007	.0000	.0000	.0000	.0000	.0000	.0000
0.5	.9606	.9415	.9127	.8681	.7977	.6877	.5177	.1249	.0102	.0005	.0000	.0000	.0000	.0000	.0000	.0000
0.6	.8981	.8651	.8210	.7610	.6787	.5667	.4142	.0969	.0079	.0004	.0000	.0000	.0000	.0000	.0000	.0000
0.8	.7300	.6823	.6262	.5594	.4799	.3855	.2724	.0616	.0050	.0003	.0000	.0000	.0000	.0000	.0000	.0000
1.0	.5710	.5231	.4700	.4109	.3449	.2715	.1886	.0419	.0034	.0002	.0000	.0000	.0000	.0000	.0000	.0000
1.2	.4463	.4036	.3578	.3087	.2555	.1991	.1369	.0301	.0025	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.5	.3162	.2826	.2476	.2111	.1731	.1333	.0909	.0199	.0016	.0001	.0000	.0000	.0000	.0000	.0000	.0000
2.0	.1932	.1709	.1483	.1252	.1018	.0778	.0527	.0114	.0009	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3.0	.0912	.0801	.0690	.0579	.0467	.0355	.0239	.0052	.0004	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4.0	.0524	.0459	.0394	.0330	.0266	.0201	.0136	.0029	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5.0	.0339	.0297	.0254	.0213	.0171	.0129	.0087	.0019	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6.0	.0237	.0207	.0177	.0148	.0119	.0090	.0061	.0013	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8.0	.0134	.0117	.0100	.0084	.0067	.0051	.0034	.0007	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
10.	.0086	.0075	.0064	.0054	.0043	.0033	.0022	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
15.	.0038	.0033	.0029	.0024	.0019	.0014	.0010	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
20.	.0022	.0019	.0016	.0013	.0011	.0008	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

TABLE XII

Single Shot Hit Probability, P(H)

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	.9998	.9928	.9606	.8981	.7300	.5710	.4463	.3162	.1932	.0912	.0524	.0339	.0237
0.1	1.000	1.000	1.000	.9995	.9905	.9557	.8919	.7247	.5675	.4441	.3151	.1927	.0911	.0524	.0339	.0237
0.2	1.000	1.000	.9999	.9975	.9816	.9397	.8730	.7090	.5571	.4375	.3117	.1915	.0908	.0523	.0338	.0236
0.3	.9998	.9995	.9975	.9879	.9606	.9104	.8415	.6839	.5405	.4270	.3062	.1894	.0904	.0522	.0338	.0236
0.4	.9928	.9905	.9816	.9606	.9224	.8667	.7985	.6512	.5188	.4130	.2989	.1865	.0897	.0519	.0337	.0236
0.5	.9606	.9557	.9397	.9104	.8667	.8106	.7464	.6128	.4932	.3963	.2900	.1830	.0889	.0517	.0336	.0235
0.6	.8981	.8919	.8730	.8415	.7985	.7464	.6888	.5710	.4650	.3776	.2797	.1788	.0879	.0513	.0334	.0234
0.8	.7300	.7247	.7090	.6839	.6512	.6128	.5710	.4852	.4054	.3370	.2566	.1690	.0855	.0505	.0331	.0233
1.0	.5710	.5675	.5571	.5405	.5188	.4932	.4650	.4054	.3477	.2959	.2319	.1579	.0825	.0494	.0326	.0230
1.2	.4463	.4441	.4375	.4270	.4130	.3963	.3776	.3370	.2959	.2574	.2075	.1462	.0792	.0482	.0321	.0228
1.5	.3162	.3151	.3117	.3062	.2989	.2900	.2797	.2566	.2319	.2075	.1738	.1286	.0737	.0461	.0311	.0223
2.0	.1932	.1927	.1915	.1894	.1865	.1830	.1788	.1690	.1579	.1462	.1286	.1020	.0641	.0422	.0293	.0213
3.0	.0912	.0911	.0908	.0904	.0897	.0889	.0879	.0855	.0825	.0792	.0737	.0641	.0468	.0339	.0250	.0190
4.0	.0524	.0524	.0523	.0522	.0519	.0517	.0513	.0505	.0494	.0482	.0461	.0422	.0339	.0266	.0208	.0165
5.0	.0339	.0339	.0338	.0338	.0337	.0336	.0334	.0331	.0326	.0321	.0311	.0293	.0250	.0208	.0171	.0140
6.0	.0237	.0237	.0236	.0236	.0236	.0235	.0234	.0233	.0230	.0228	.0223	.0213	.0190	.0165	.0140	.0119
8.0	.0134	.0134	.0134	.0134	.0134	.0133	.0133	.0133	.0132	.0131	.0129	.0126	.0117	.0107	.0096	.0086
10.	.0086	.0086	.0086	.0086	.0086	.0086	.0086	.0085	.0085	.0085	.0084	.0083	.0079	.0074	.0069	.0063
15.	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0038	.0037	.0036	.0034	.0033
20.	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0022	.0021	.0021	.0021	.0021	.0021	.0021	.0020	.0020

TABLE XIII
Probability of Achieving Exactly Two Hits of Two Rounds Fired, P{HH}

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION μ IN MILS

	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	.9996	.9856	.9228	.8065	.5329	.3261	.1992	.1000	.0373	.0083	.0027	.0011	.0006
0.1	1.000	1.000	1.000	.9991	.9812	.9133	.7954	.5252	.3221	.1972	.0993	.0372	.0083	.0027	.0011	.0006
0.2	1.000	1.000	.9998	.9951	.9638	.8835	.7628	.5031	.3106	.1915	.0972	.0367	.0083	.0027	.0011	.0006
0.3	.9998	.9992	.9954	.9769	.9248	.8318	.7114	.4697	.2930	.1826	.0939	.0359	.0082	.0027	.0011	.0006
0.4	.9928	.9866	.9695	.9303	.8603	.7615	.6469	.4293	.2715	.1715	.0896	.0348	.0081	.0027	.0011	.0006
0.5	.9606	.9432	.9100	.8551	.7770	.6807	.5767	.3860	.2479	.1591	.0847	.0336	.0079	.0027	.0011	.0006
0.6	.8981	.8697	.8258	.7644	.6869	.5987	.5073	.3433	.2241	.1461	.0793	.0322	.0077	.0026	.0011	.0005
0.8	.7300	.6916	.6431	.5854	.5210	.4531	.3858	.2671	.1798	.1209	.0683	.0290	.0073	.0026	.0011	.0005
1.0	.5710	.5335	.4906	.4434	.3935	.3428	.2936	.2075	.1433	.0989	.0579	.0258	.0069	.0025	.0011	.0005
1.2	.4463	.4134	.3777	.3402	.3018	.2636	.2268	.1628	.1147	.0809	.0489	.0227	.0064	.0023	.0010	.0005
1.5	.3162	.2907	.2642	.2374	.2106	.1845	.1597	.1165	.0839	.0606	.0379	.0186	.0056	.0022	.0010	.0005
2.0	.1932	.1764	.1597	.1433	.1272	.1118	.0973	.0721	.0530	.0392	.0256	.0134	.0045	.0019	.0009	.0005
3.0	.0912	.0829	.0749	.0671	.0596	.0526	.0459	.0345	.0259	.0196	.0132	.0074	.0029	.0013	.0007	.0004
4.0	.0524	.0476	.0429	.0384	.0342	.0302	.0264	.0200	.0151	.0115	.0079	.0046	.0019	.0009	.0005	.0003
5.0	.0339	.0307	.0277	.0248	.0221	.0195	.0171	.0129	.0098	.0075	.0052	.0031	.0013	.0007	.0004	.0002
6.0	.0237	.0215	.0193	.0173	.0154	.0136	.0119	.0090	.0069	.0053	.0037	.0022	.0010	.0005	.0003	.0002
8.0	.0134	.0121	.0109	.0098	.0087	.0077	.0067	.0051	.0039	.0030	.0021	.0013	.0006	.0003	.0002	.0001
10.	.0086	.0078	.0070	.0063	.0056	.0049	.0043	.0033	.0025	.0019	.0014	.0008	.0004	.0002	.0001	.0001
15.	.0038	.0035	.0031	.0028	.0025	.0022	.0019	.0015	.0011	.0009	.0006	.0004	.0002	.0001	.0001	.0000
20.	.0022	.0020	.0018	.0016	.0014	.0012	.0011	.0008	.0006	.0005	.0003	.0002	.0001	.0001	.0000	.0000

TABLE XIV
Average Number of Rounds to Achieve a Hit

ROUND TO ROUND STANDARD DEVIATION ν IN MILS

TOTAL BIAS STANDARD DEVIATION η IN MILS	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2	1.5	2.0	3.0	4.0	5.0	6.0
0.0	1.000	1.000	1.000	1.000	1.007	1.041	1.113	1.370	1.751	2.241	3.162	5.177	10.96	19.07	29.50	42.24
0.1	-	1.000	1.000	1.000	1.010	1.046	1.121	1.380	1.762	2.252	3.174	5.188	10.97	19.08	29.51	42.26
0.2	-	-	1.000	1.003	1.019	1.065	1.146	1.412	1.796	2.287	3.209	5.223	11.01	19.12	29.54	42.29
0.3	-	-	-	1.014	1.044	1.103	1.195	1.469	1.856	2.347	3.269	5.283	11.07	19.17	29.60	42.35
0.4	-	-	-	-	1.105	1.178	1.278	1.560	1.948	2.437	3.357	5.368	11.15	19.26	29.68	42.43
0.5	-	-	-	-	-	1.331	1.425	1.700	2.081	2.565	3.477	5.482	11.26	19.36	29.79	42.54
0.6	-	-	-	-	-	-	1.723	1.922	2.274	2.740	3.637	5.629	11.39	19.50	29.92	42.66
0.8	-	-	-	-	-	-	-	3.150	3.015	3.329	4.12'	6.039	11.76	19.84	30.26	43.00
1.0	-	-	-	-	-	-	-	-	5.789	4.655	4.979	6.665	12.26	20.30	30.70	43.44
1.2	-	-	-	-	-	-	-	-	-	10.20	6.706	7.632	12.93	20.90	31.27	43.98
1.5	-	-	-	-	-	-	-	-	-	-	21.69	10.43	14.38	22.09	32.36	45.02
2.0	-	-	-	-	-	-	-	-	-	-	-	61.36	18.99	25.19	35.00	47.45
3.0	-	-	-	-	-	-	-	-	-	-	-	-	286.6	42.09	45.63	56.09
4.0	-	-	-	-	-	-	-	-	-	-	-	-	-	879.3	79.43	75.29
5.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2108.	134.5
6.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4286.
8.0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

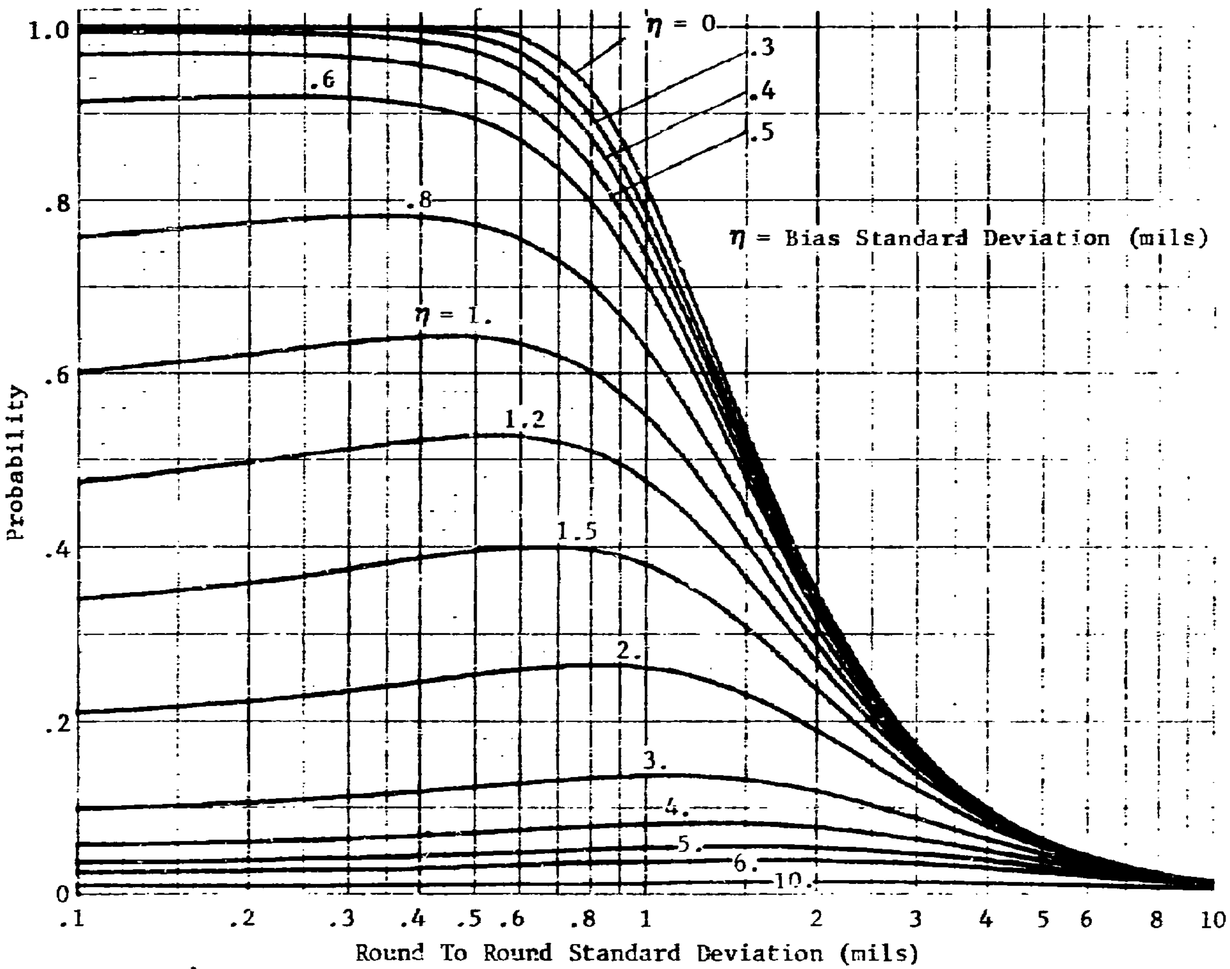

Figure 1. Probability of Achieving at Least One Hit of a Two Round Burst

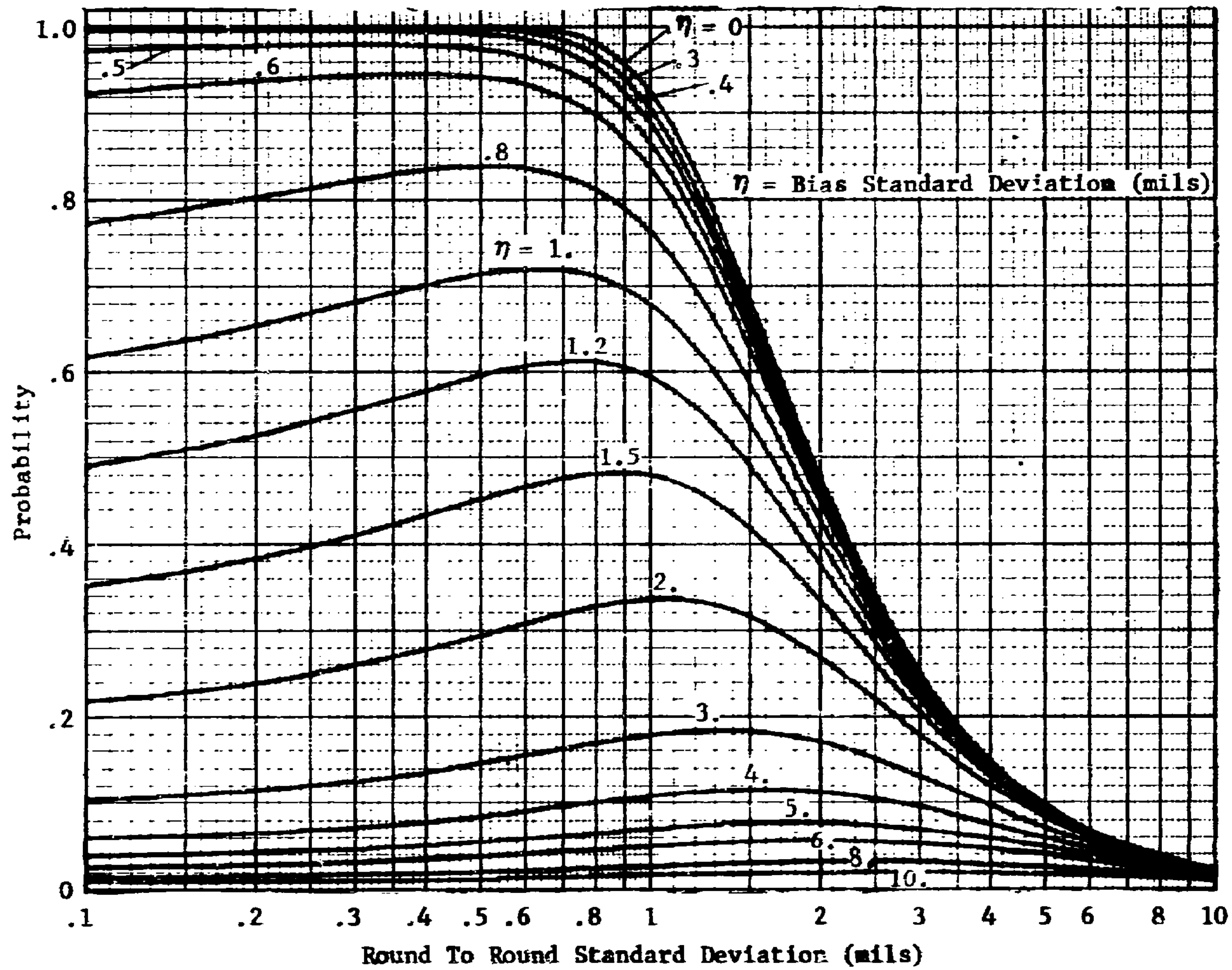

Figure 2. Probability of Achieving at Least One Hit of a Three Round Burst

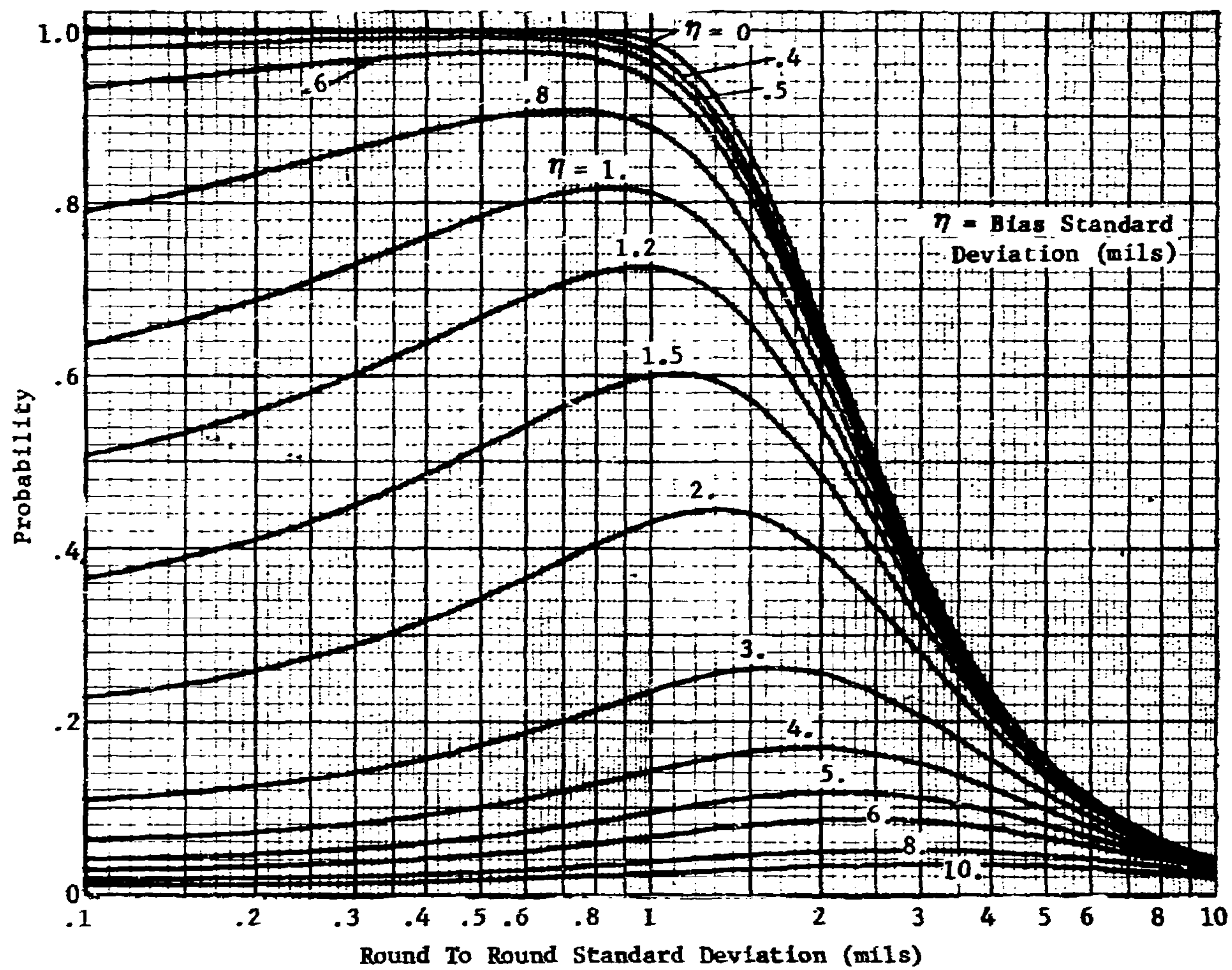

Figure 3. Probability of Achieving at Least One Hit of a Five Round Burst

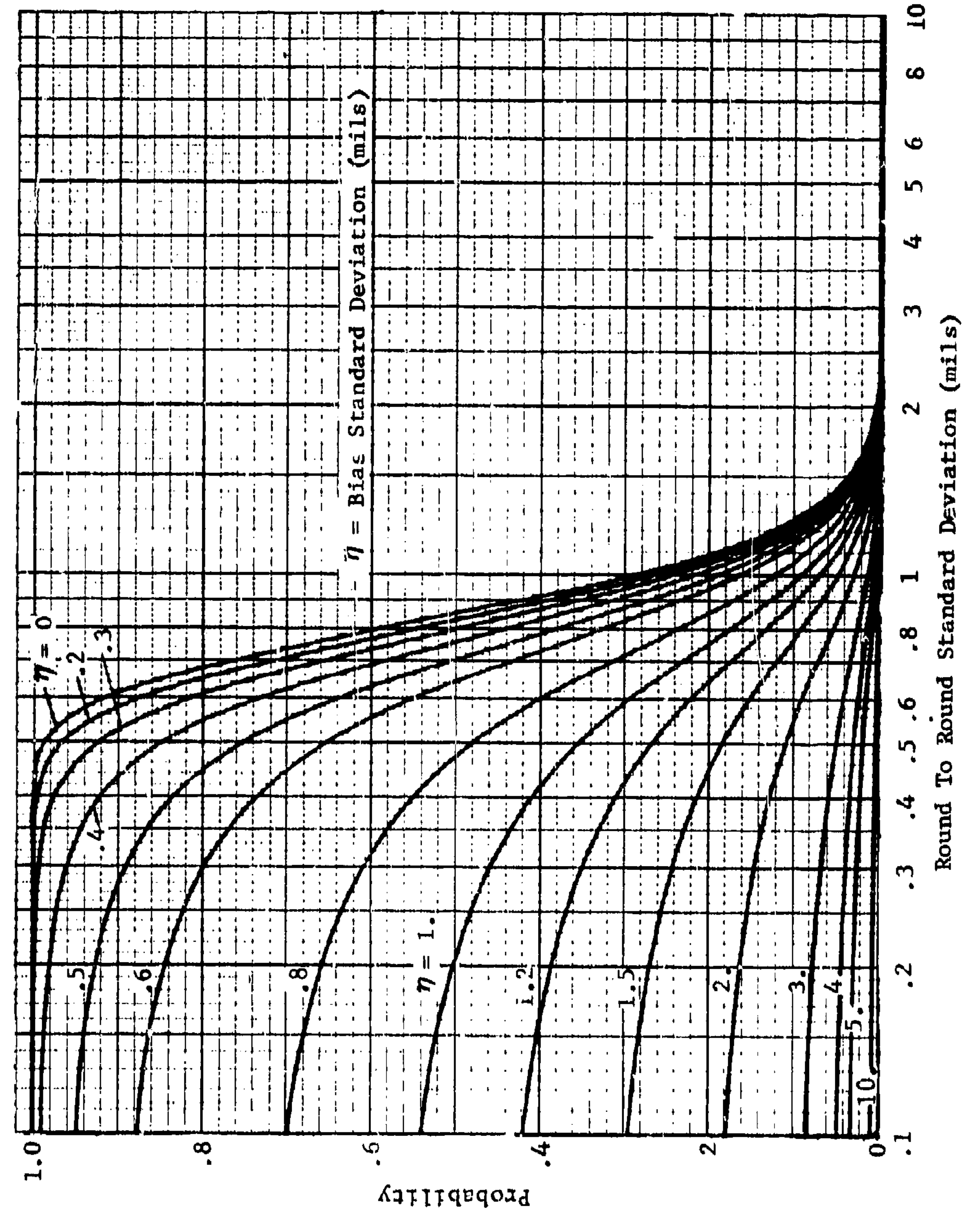

Figure 4. Probability of Achieving at Least Four Hits of a Five Round Burst

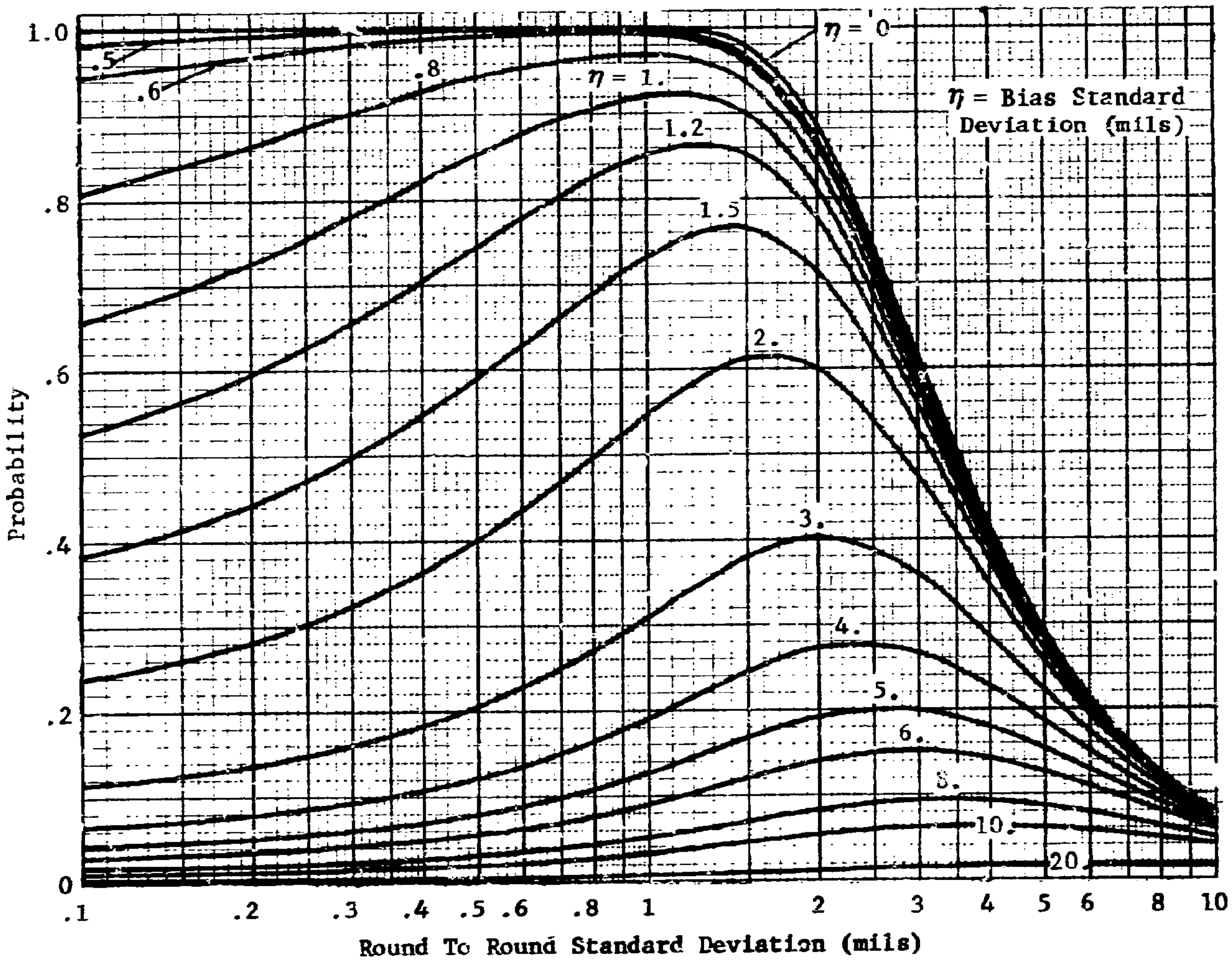

Figure 5. Probability of Achieving at Least One Hit of a Ten Round Burst

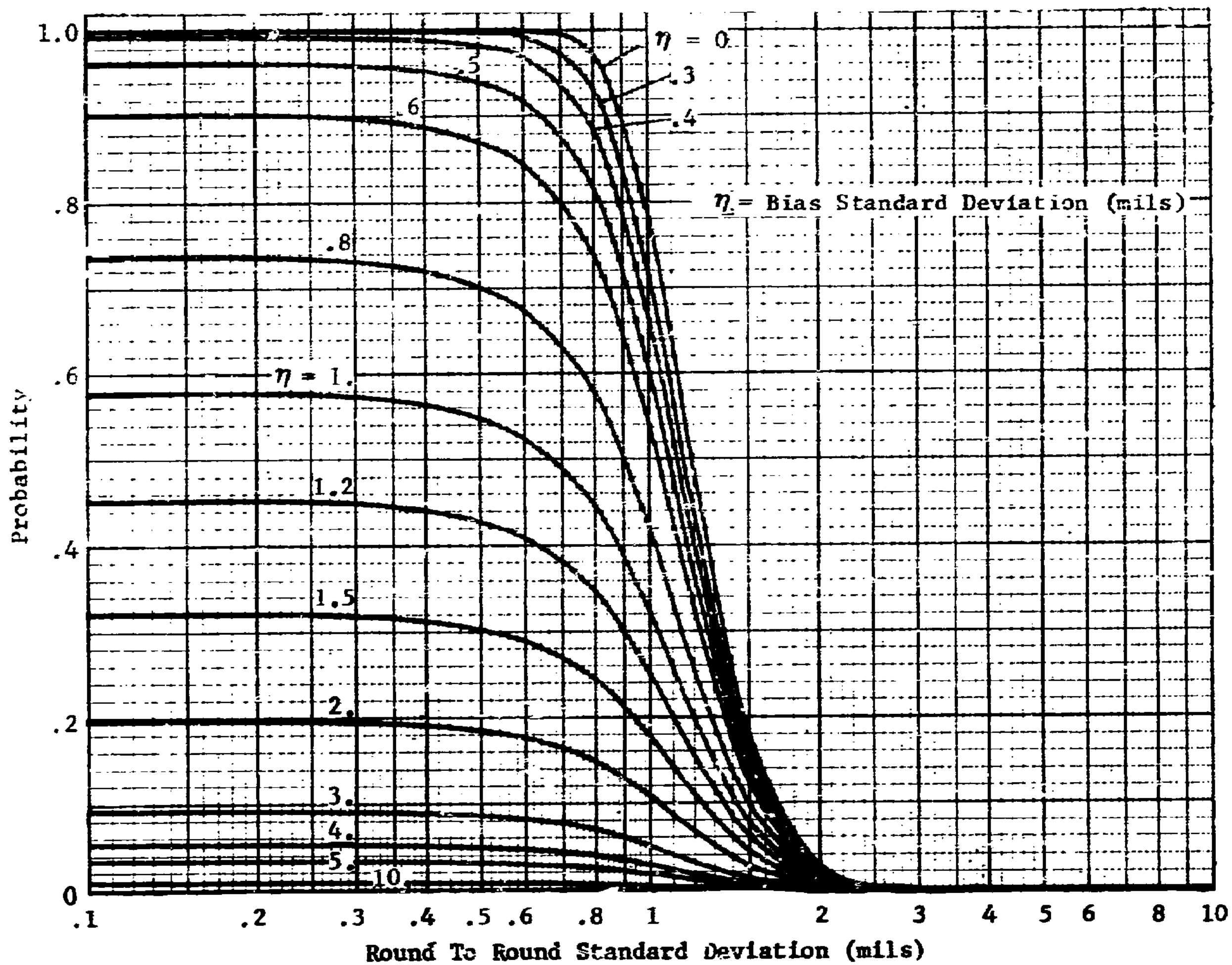

Figure 6. Probability of Achieving at Least Five Hits of a Ten Round Burst

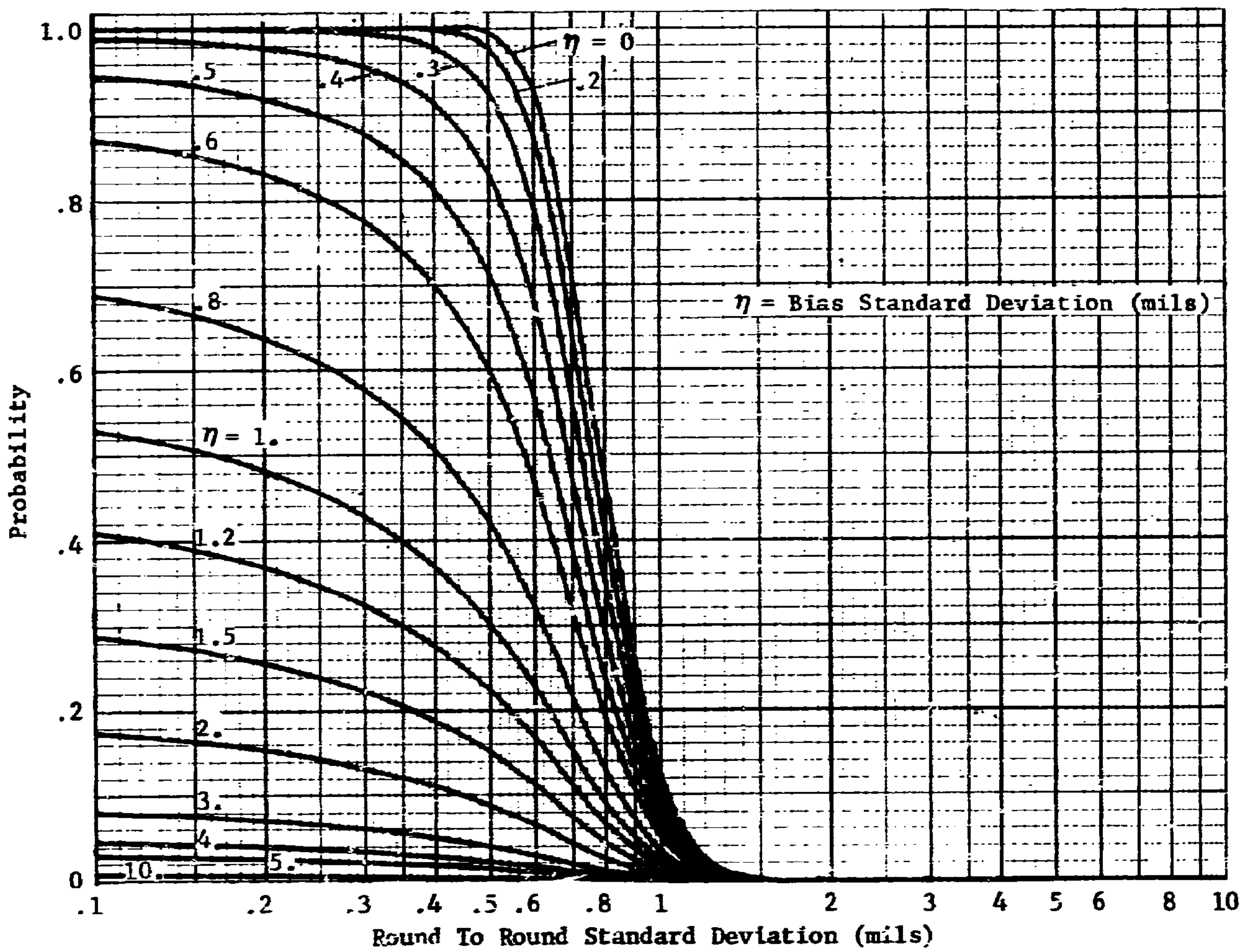

Figure 7. Probability of Achieving at Least Eight Hits of a Ten Round Burst

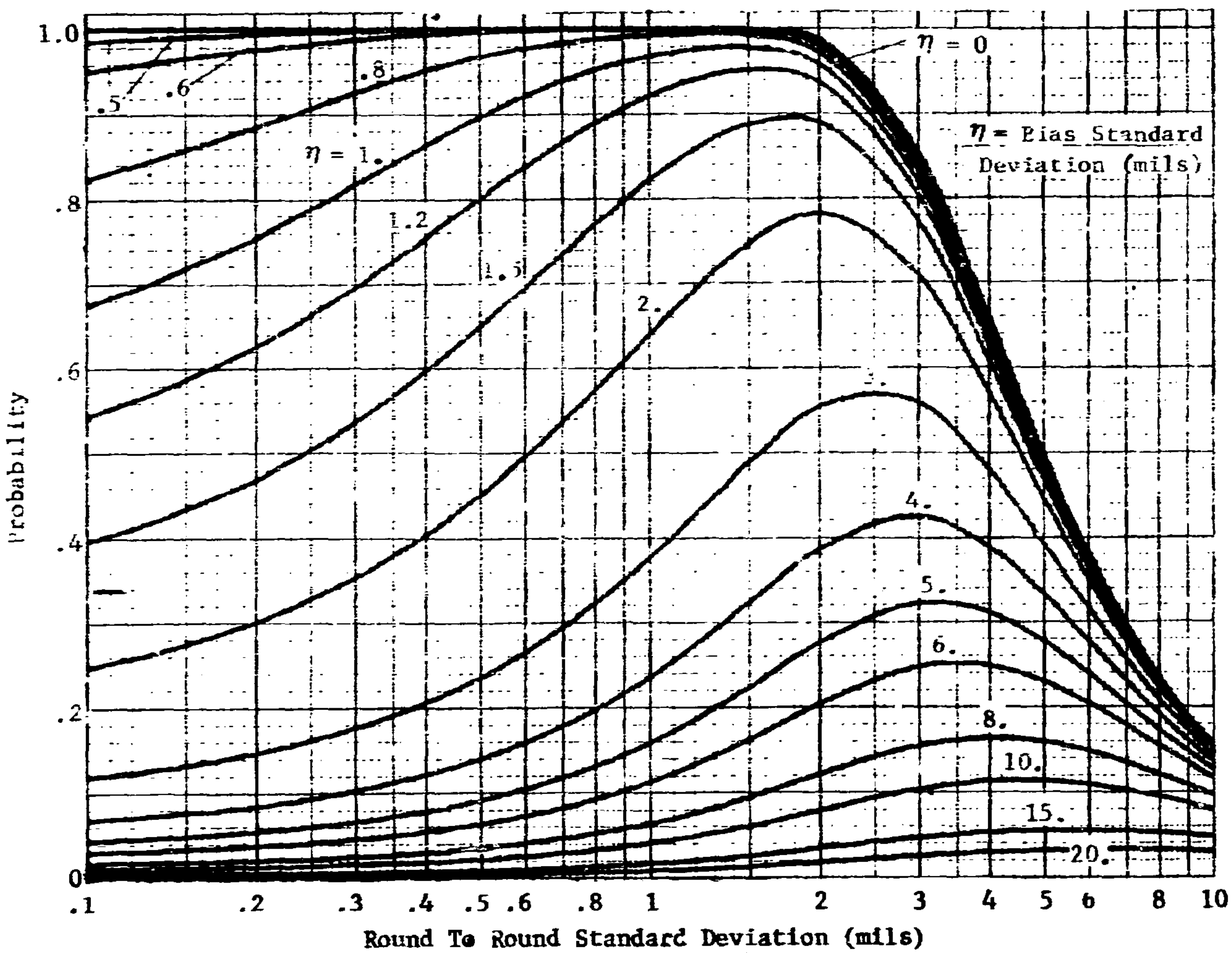

Figure 8. Probability of Achieving at Least One Hit of a Twenty Round Burst

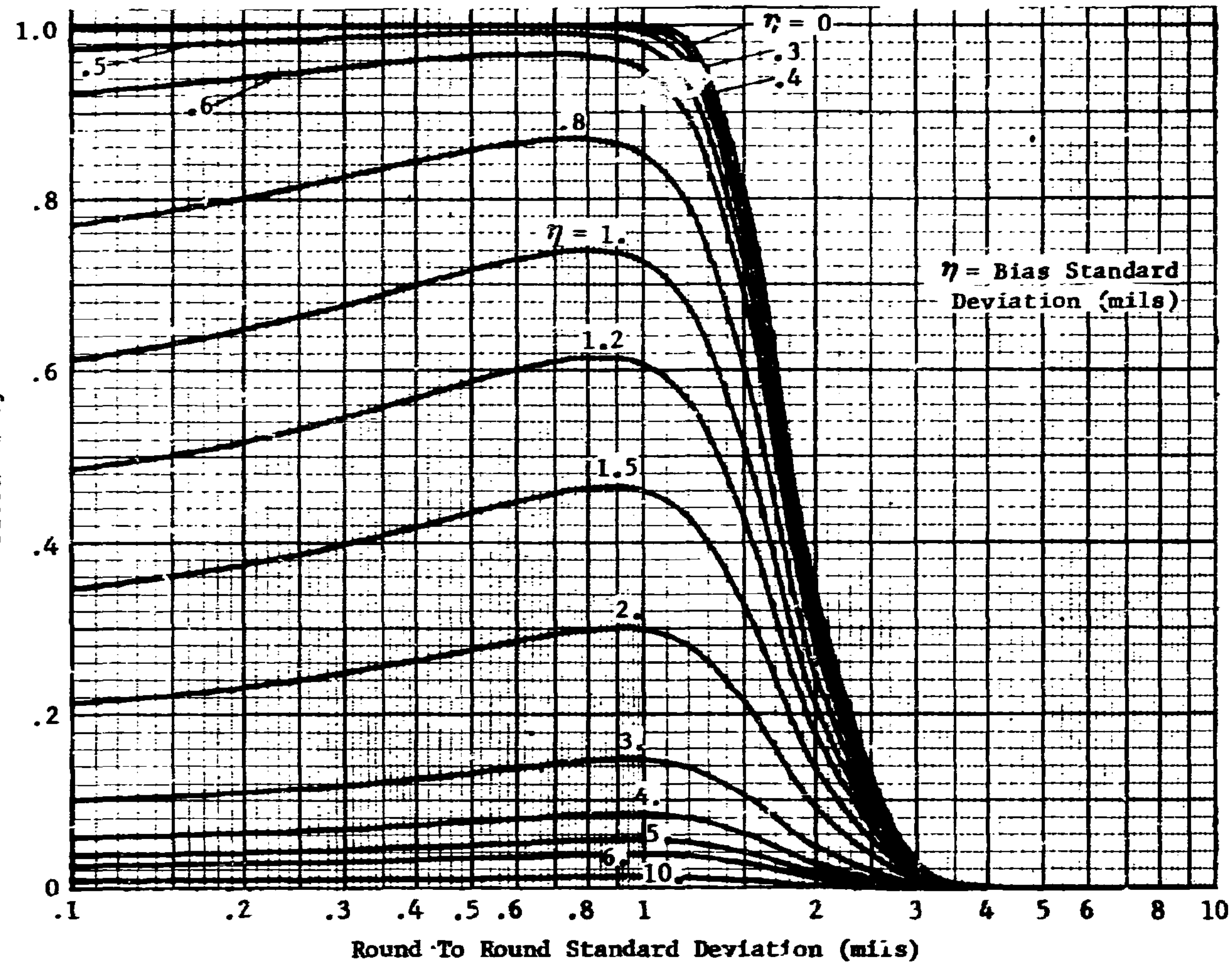

Figure 9. Probability of Achieving at Least Five Hits of a Twenty Round Burst

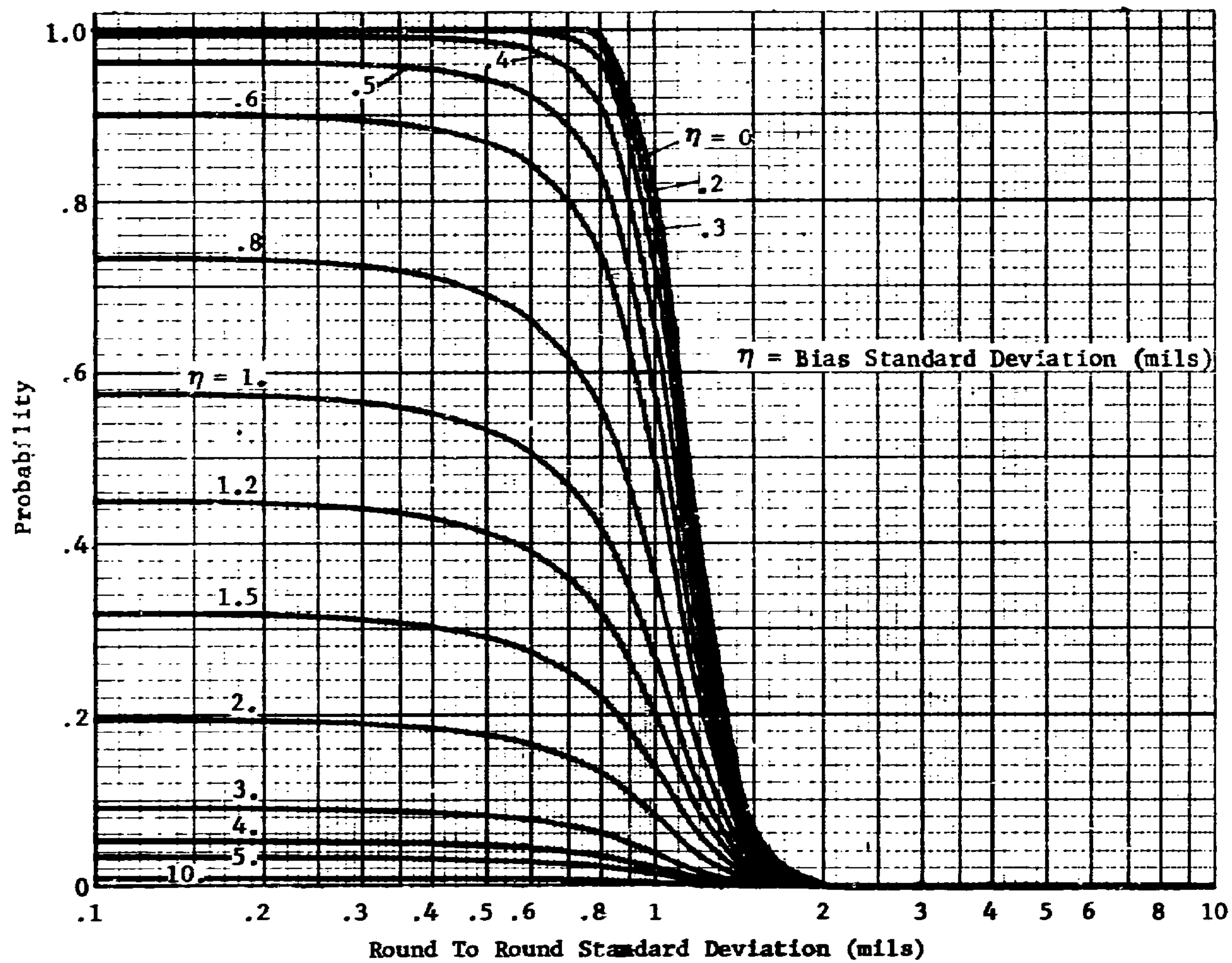

Figure 10. Probability of Achieving at Least Ten Hits of a Twenty Round Burst

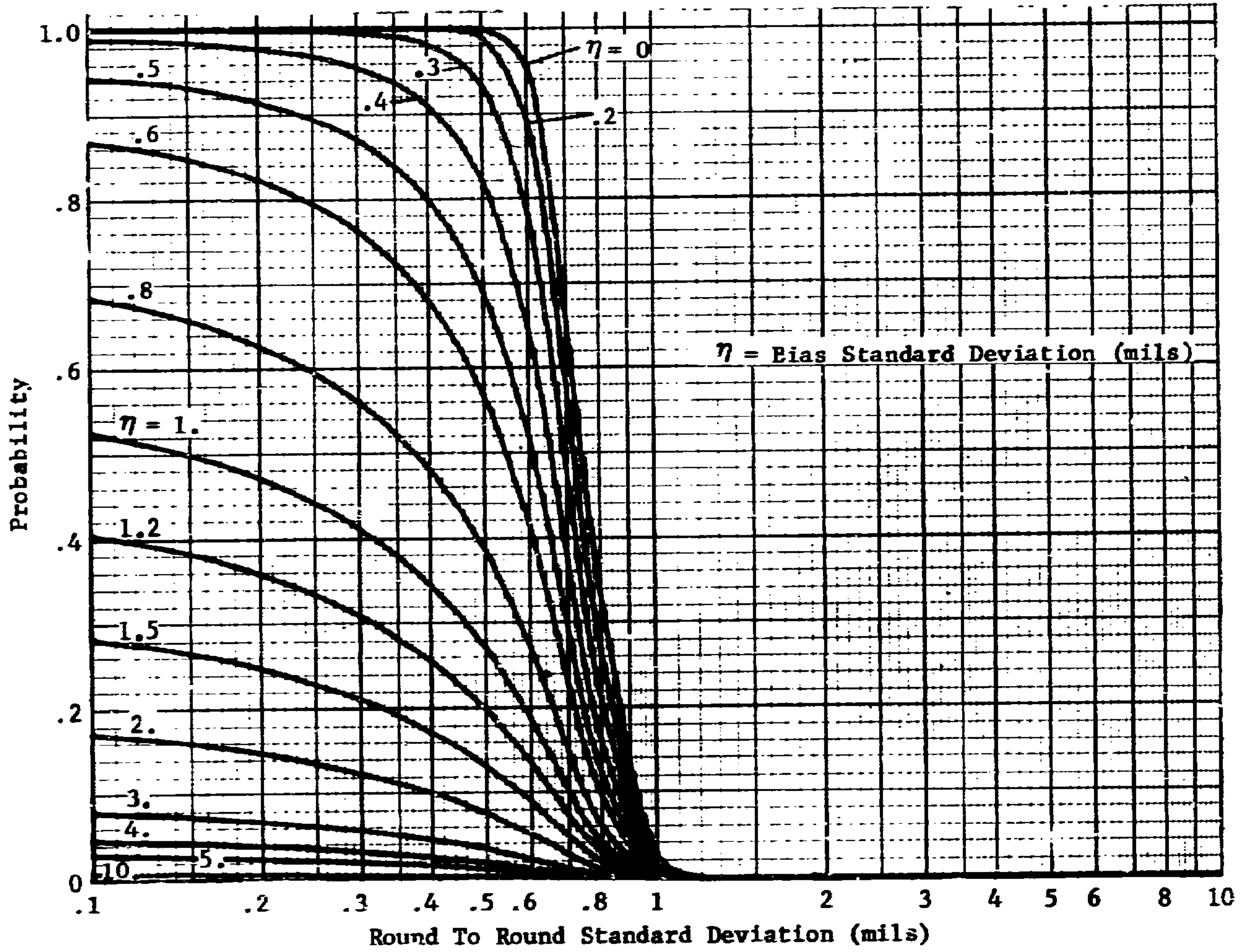

Figure 11. Probability of Achieving at Least Sixteen Hits of a Twenty Round Burst

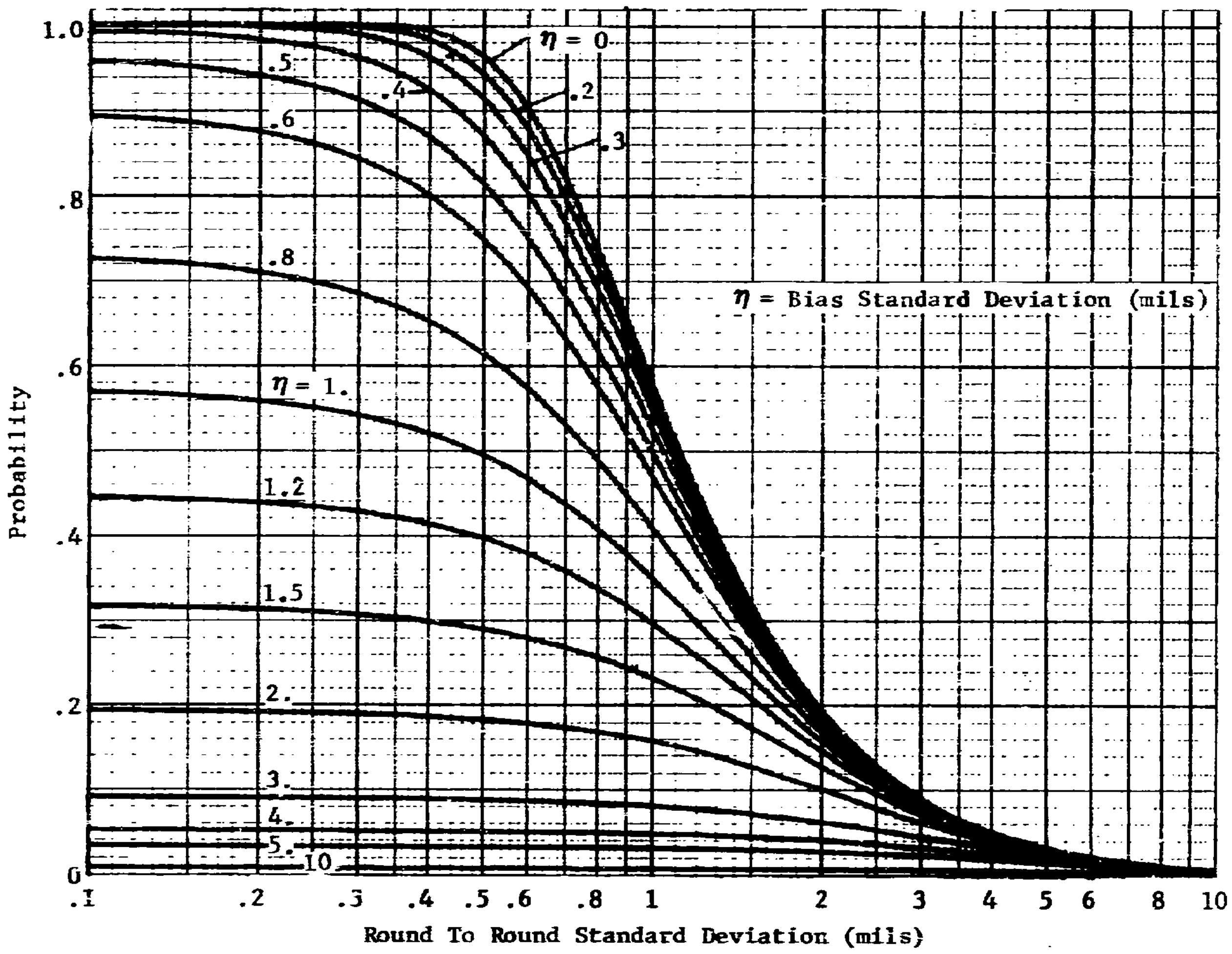

Figure 12. Single Shot Hit Probability, P{H}

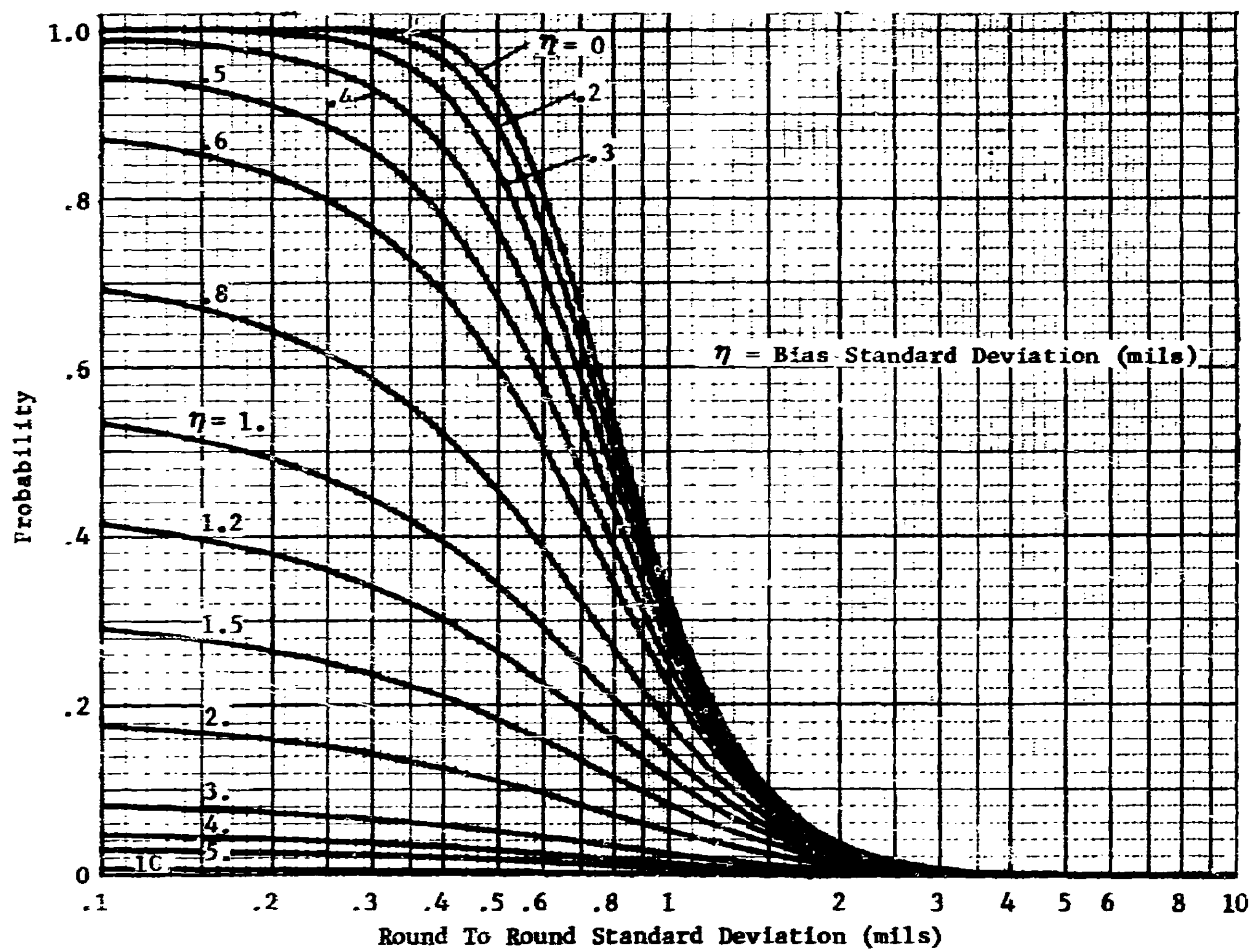

Figure 13. Probability of Achieving Exactly Two Hits of Two Rounds Fired, P{HH}

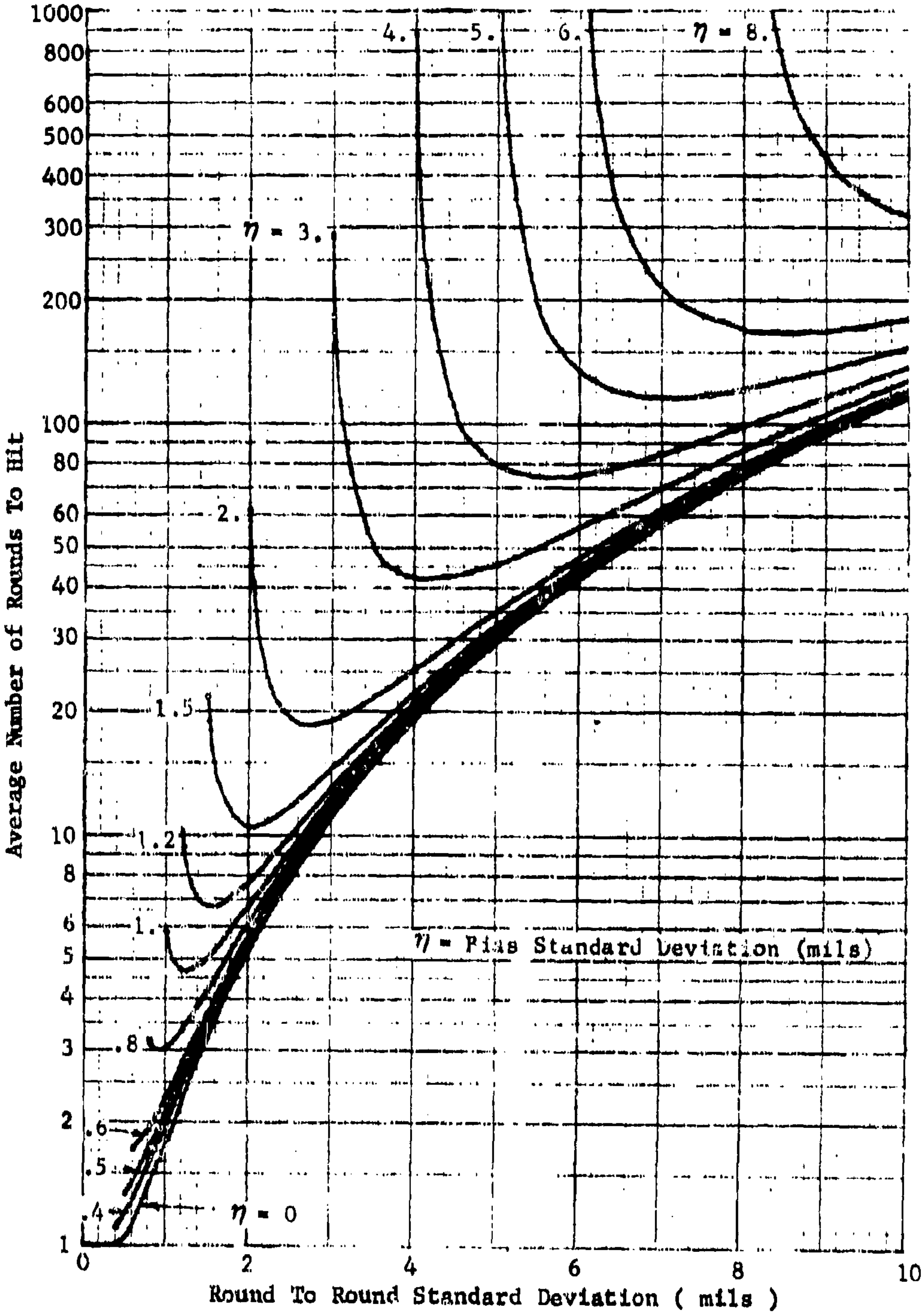

Figure 14. Average Number of Rounds to Achieve a Hit

GLOSSARY OF SYMBOLS

The symbols used in this report were chosen to permit reference to other texts on the subject and to maintain consistency throughout the report itself.

GREEK LETTERS

μ_x, μ_y	The fixed bias in the horizontal and vertical dimensions respectively.
γ_x^2, γ_y^2	The variances of the occasion-to-occasion biases in the horizontal and vertical dimensions respectively.
ψ_x^2, ψ_y^2	The variances of the burst-to-burst biases in the horizontal and vertical dimensions respectively.
η_x^2, η_y^2	The variances of the total bias present during a burst in the horizontal and vertical dimensions respectively.
ν_x^2, ν_y^2	The variances of the round-to-round errors in the horizontal and vertical dimensions respectively.

PROBABILITIES

$P\{H\}$	First round hit probability.
$P\{K/N\}$	The probability of obtaining exactly K hits on a target from a sequence of N rounds fired.
$P\{H\|x,y\}$	The conditional probability of a hit given the values x and y, horizontal and vertical components of bias respectively.
$P\{K/N\|x,y\}$	The conditional probability of obtaining exactly K hits of N rounds fired given the values x and y, the horizontal and vertical components of bias respectively.
$P\{K_1/N_1, K_2/N_2\}$	The probability of obtaining exactly K_1 hits from an N_1 round first burst <u>and</u> exactly K_2 hits from an N_2 round second burst.

OTHER TERMS

E_R	The expected or average number of rounds which must be fired to obtain a first hit.
E_H	The expected or average number of hits obtained when firing a burst.
σ_H^2	The variance of the number of hits obtained when firing a burst.